THE PRINCETON COLLOQUIUM.

THE Sixth Colloquium of the American Mathematical Society was held, at the close of the sixteenth summer meeting, at Princeton University, Princeton, N. J., opening on Wednesday morning, September 15, 1909, and extending until the following Friday.* At the April meeting, 1908, the Council appointed a committee consisting of Professors H. B. Fine, W. F. Osgood, T. F. Holgate and F. N. Cole to arrange for the colloquium. A preliminary circular announcing the general features was issued in May of 1909. The colloquium opened on Wednesday morning, September 15, 1909, in the lecture room of McCosh Hall, Princeton University, the following 28 persons being in attendance:

Professor G. D. Birkhoff, Professor G. A. Bliss, Mr. R. D. Carmichael, Dr. A. B. Chace, Dr. A. Cohen, Dr. G. M. Conwell, Dr. L. S. Dederick, Professor L. P. Eisenhart, Professor T. C. Esty, Professor H. B. Fine, Dr. G. F. Gundelfinger, Dr. Frank Irwin, Professor Edward Kasner, Mr. A. K. Krause, Professor W. R. Longley, Mr. H. H. Mitchell, Professor J. H. Maclagan-Wedderburn, Mr. H. F. MacNeish, Professor E. H. Moore, Professor Frank Morley, Professor G. D. Olds, Mr. W. J. Risley, Professor Virgil Snyder, Mr. C. E. Van Orstrand, Professor E. B. Van Vleck, Professor Oswald Veblen, Professor H. S. White, Professor J. E. Wright.

Two courses of lectures were given, as follows:

I. Professor G. A. BLISS: "Fundamental existence theorems." Four lectures.

II. Professor EDWARD KASNER: "Geometric aspects of dynamics." Four lectures.

Two lectures were given each morning, one on Wednesday afternoon, and one at noon on Friday, the lecturers alternating. Printed syllabi of both courses had been issued in advance, as usual. Thursday afternoon was devoted to an excursion to Washington's Headquarters, carriages and automobiles being provided by Dean Fine and Mr. Pyne, of the University. The evenings were spent in social conference at the Princeton

* For the history of the preceding colloquia, see the Report of the New Haven Colloquium, BULLETIN, vol. 13 (1906–07), page 71, where complete references are given.

Inn, the general headquarters of the meeting. The hospitality of Princeton University, and particularly of the mathematical department, was gratefully acknowledged by a vote of thanks at the closing meeting.

The following abstracts of the lectures convey a general idea of their content. More detailed reports by the lecturers will appear in later numbers of the BULLETIN.

I. The earlier part of Professor Bliss's course was devoted to a review of the theory of implicit functions, including a detailed account of some of the recent developments in the subject, with their applications in the calculus of variations. The existence theorems for ordinary differential equations were taken up with special reference to the definition of solutions over an extended region and their behavior as functions of the initial constants. A short account of the geometrical theory of partial differential equations of the first order was given, and with this as a guide the results already obtained were applied to show the existence of solutions of such equations, even when the function defining the equation is not analytic. The theory of implicit functions for real variables and some knowledge of the methods of approximation of Cauchy and Picard for ordinary differential equations were presupposed.

II. All physical phenomena take place in space and may therefore suggest geometric investigation. In this connection the attention given to kinetics has been slight in comparison with that devoted to statics and kinematics. Professor Kasner's lectures dealt with geometric aspects of kinetics. Such topics as the following were treated:

Conservative systems, the principle of least action, reduction to geodesics, Thomson's theorem, natural families, conformal transformations (Larmor, Goursat, Darboux). General fields of force, trajectories, geometric explorations. Projective properties and Appell's transformation. Interrelations of catenaries, brachistochrones, and tautochrones. Representation of time (Minkowski); representation of phase in statistical mechanics. Transformations of time, contact transformations. Problem of several bodies. Non-holonome systems and Hertz's geometry of material systems. Optics and elasticity.

VIRGIL SNYDER.

THE SEPTEMBER MEETING OF THE SAN FRANCISCO SECTION.

THE sixteenth regular meeting of the San Francisco Section of the American Mathematical Society was held at the University of California, Saturday, September 25, 1909. The following members were present:

Professor R. E. Allardice, Professor H. F. Blichfeldt, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor A. O. Leuschner, Professor H. C. Moreno, Professor C. A. Noble, Mr. E. W. Ponzer, Mr. H. W. Stager, Professor A. L. Whitney.

The following officers were elected for the ensuing year: Professor Blichfeldt, Chairman; Professor Noble, Secretary; Professors Hoskins, Lehmer, Noble, Program Committee. The next two meetings are to be held respectively at Stanford University February 26, 1910, and at the University of California September 24, 1910.

The following papers were read at this meeting:

(1) Professor D. N. LEHMER: "On the arithmetical theory of pencils of binary quadratic forms" (preliminary communication).

(2) Professor H. F. BLICHFELDT: "On the infinitude of primes in certain arithmetical progressions" (preliminary communication).

(3) Mr. G. F. MCEWEN: "The motion of a viscous fluid between two parallel planes" (preliminary communication).

(4) Professor L. M. HOSKINS: "The strain of a gravitating compressible elastic sphere."

(5) Professor A. O. LEUSCHNER: "An equation giving the geocentric distance in the problem of determining parabolic orbits from geocentric observations."

Mr. McEwen was introduced by Professor Blichfeldt.

Abstracts of the papers are given below in order as numbered in the foregoing list:

1. Professor Lehmer gives a study of the arithmetical properties of the forms included in the pencil $\alpha A + \beta B$, where α and β are integers and A and B binary quadratic forms. The

forms of the pencil must have determinants representable by a certain binary quadratic form $H = Dx^2 + \Theta xy + D'y^2$, where D and D' are the determinants of A and B and Θ is the joint invariant. If $A = (abc)$, $B = (a'b'c')$, the form $J = (ab' - a'b)x^2 + (ac' - a'c)xy + (bc' - b'c)y^2$ is also of fundamental importance in the theory. A pencil may be found having a given form J. A pencil may or may not be found having a given form H, according as H is or is not of the principal genus. The form H is the duplicate of J if J is a primitive form.

2. By elementary algebraic processes, involving the approximate evaluation of certain factorials, Professor Blichfeldt proved that the arithmetical progressions k, $11 + k$, $22 + k$, $33 + k$, $\cdots$ (k prime to 11) contain an infinite number of primes each.

3. Mr. McEwen's paper is in abstract as follows: A viscous fluid is confined between two parallel planes, one being fixed, the other having a displacement in its own plane,

$$x_1 = ae^{-\alpha t} \sin \sigma t.$$

Assuming the distance between the planes to be great, the displacement of the fluid at the distance y from the moving plane is

$$x_2 = ae^{-\alpha t - \beta y} \sin (\sigma t - \beta y),$$

where $\beta = \sqrt{\sigma\rho/2\mu}$, $\rho =$ density of the fluid, $\mu =$ the coefficient of viscosity of the fluid.

A gravity pendulum is hung so that a small plane attached to its lower end is parallel to the plane of vibration of the pendulum and the fluid in which it is immersed. x_3 is the displacement of this plane. x_1', x_2', and x_3' are the maximum values of x_1, x_2, and x_3.

$$\frac{x_2'}{x_1'} = e^{-\beta\left(1+\frac{\alpha}{\sigma}\right)y} \quad \text{and} \quad \frac{x_3'}{x_1'} = \left(1 - \frac{\alpha^2}{2\sigma^2}\right) e^{-\beta\left(1+\frac{\alpha}{\sigma}\right)y},$$

if $x_3 = ae^{-\alpha t} \sin \sigma t$ when the plane is not in the fluid and if α/σ is small.

4. In Kelvin's well-known solution of the problem of the strain of an elastic sphere, the bodily forces are assumed to be known functions of the coordinates of position. When self-

gravitation is considered this solution is inapplicable, except in the case of incompressibility, because the force of attraction acting upon any volume element depends in part upon the change of density produced in that element by the strain and upon the change of density distribution of the attracting mass. A solution of the problem taking account of the actual gravitational forces in the strained configuration is given in Professor Hoskins's paper. The problem is worked out completely for the case in which the strain is due to disturbing forces of the type of tidal or centrifugal forces, and numerical results have been obtained corresponding to several different values of the ratio of the elastic constants. The strain at any distance from the center being specified by two quantities — the ellipticity of the originally spherical surface and the angular displacement of a radius vector inclined 45° to the axis of symmetry — it is found that for a given value of the rigidity modulus, the former of these quantities is decreased and the latter increased by compressibility. The solution has also been generalized so as to apply to the case in which the potential of the disturbing forces is any spherical harmonic of degree not less than 2.

5. In his adaptation of the "short method of determining orbits" to the direct computation of a parabola for comets, Professor Leuschner derives the geocentric distance ρ at a fixed date from the equation

$$(z-p')^2-\frac{n}{[(z-c)^2+s^2]^{\frac{1}{2}}}-q'^2=0,$$

where $z=\rho/R$, and R is the distance of the sun. p', n, q'^2, c, and s are auxiliary quantities depending on observed coordinates and other data. c and s are the sine and cosine of the angle ψ subtended at the observer by the arc between the comet and the sun.

The solutions are given by the intersections of the parabola $y=z'^2$ and the curve $y=n/\sqrt{[z'-c']^2+s^2}-q'^2$ for which z' is real and positive, where $z'=z-p'$ and $c'=c-p'$.

There will be either one or three solutions. Three solutions exist, if
either

$$p'>0, \quad c>0;$$

or

$$p'>0, \quad c<0, \quad 90°<\psi<125°\ 16';$$

or

$$p' < 0, \quad c > 0, \quad 0° < \psi < 54° \; 44',$$

and if

$$\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2 < 0,$$

where

$$\frac{p}{3} = \frac{1}{81}\left[9(2s^2 + q'^2) - 7c'^2\right]; \quad \frac{q}{2} = \frac{5c'}{9}\left[\frac{p}{3} + \frac{1}{9}\left(m^2 - \frac{11}{10}q'^2\right)\right]$$

and

$$m^2 = c'^2 + s^2.$$

For solution the equation in z is written in the form

$$y' = (\zeta + c')^2 - (\eta - q'^2) = f(\vartheta) = 0,$$

where

$$\zeta = s \tan \vartheta; \quad \eta = \frac{n}{s} \cos \vartheta.$$

A convenient graphical solution is proposed for the solution of $f(\vartheta) = 0$. Then

$$\rho/R = z = s \tan \vartheta + c.$$

Geocentric distances correct to four or five decimals result from the graphical solution. Further decimals may be obtained by a simple differential correction.

In practice no case with three solutions has been encountered.

C. A. NOBLE,
Secretary of the Section.

THE WINNIPEG MEETING OF THE BRITISH ASSOCIATION.

THE seventy-ninth annual meeting of the British Association for the advancement of science was held in Winnipeg August 25 to September 1. Fourteen hundred members and associates were in attendance. The opening event was the address of the President of the Association, Sir J. J. Thomson, on Wednesday evening, August 25, in which he gave an account of some of the more recent developments in physics and in his opening remarks took occasion to urge a closer union between mathematics and physics and to emphasize the advantages of

research in developing the qualities of a student. The following morning the sectional meetings opened with the addresses of the presidents of the various sections. In Section A President Rutherford took as his subject the present position of the atomic theory in physical science. The sectional addresses, as also a more general account of the meeting of the Association, will be found in *Nature* and in *Science;* for abstracts of the various papers presented the reader is referred to the annual Report of the proceedings of the Association. Section A covers the mathematical and physical sciences and the Winnipeg meeting of the section was a particularly successful one, in which however the subject of physics was dominant. There were six sessions in physics with crowded programmes and a large number in attendance. Cosmical physics, included in the Section for convenience, occupied several separate sessions. One session sufficed for the presentation of the papers in pure mathematics, which were the following:

(1) E. H. MOORE: "Theorems in general analysis."

(2) E. W. HOBSON: "On the present position of the theory of aggregates."

(3) G. A. MILLER: "Generalizations of the icosahedral group."

(4) G. A. BLISS: "A new proof of Weierstrass's factor theorem."

(5) J. H. GRACE: "On ideal numbers."

(6) P. A. MACMAHON: "On a correspondence in the theory of the partition of numbers."

(7) W. H. METZLER: "On a continuant expressed as the product of linear factors."

(8) E. W. DAVIS: "Imaginary geometry of the conic."

(9) F. CAJORI: "On the invention of the slide rule."

(10) J. W. NICHOLSON: "The asymptotic expansion of Legendre functions."

(11) Report on the calculation of Bessel functions.

1. Professor Moore started out from the statement of the fundamental principle: "The existence of analogies between central features of various theories implies the existence of a general theory which underlies the particular theories and unifies them with respect to those central features." The speaker emphasized the importance of such a general theory for the subject of integral equations. He also considered a number of impor-

tant analytic systems included under a certain general type, with reference to the properties which they had in common.

2. Professor Hobson recalled various points which have been raised in recent controversies relative to the theory of aggregates, and advocated the adoption of a definition of an aggregate of more restricted character than that of G. Cantor.

3. The general groups treated in Professor Miller's paper are those in which the two generating operators t_1, t_2 satisfy one of the three sets of conditions

$$t_1^2 = t_2^5, \quad (t_1t_2)^3 = (t_2t_1)^3; \quad t_1^2 = t_2^3, \quad (t_1t_2)^5 = (t_2t_1)^5;$$
$$t_1^3 = t_2^5, \quad (t_1t_2)^2 = (t_2t_1)^2.$$

Among others Professor Miller established the following important theorems: There are an infinite number of groups each of which may be generated by two operators satisfying one of these pairs of conditions. Each of the possible groups generated by t_1, t_2 contains either the icosahedral group or the group of order 120 which is unsolvable and does not contain a subgroup of order 60, and it must have one of these groups for its commutator subgroup.

4. The theorem of Weierstrass for which Professor Bliss has found a simplified proof is the following: Any convergent series in $p + 1$ variables $F(x_1, x_2, \cdots, x_p, y)$ in which the lowest term of y alone is of degree n, can be expressed as a product

$$(y^n + a_1y^{n-1} + \cdots + a_{n-1}y + a_n)\phi(x_1, \cdots, x_p, y),$$

where $a_1, \cdots, a_n$ are convergent series in $x_1, \cdots, x_p$ which vanish with these arguments, while ϕ is a convergent series with a constant term different from zero, in all $p + 1$ variables.

5. Dr. Grace's paper treated of the ideals in a field defined by the root of a quadratic equation.

7. Professor Metzler's theorem includes as special cases theorems due to Painvin, Sylvester, and Cayley, and may be stated thus:

$$|x_{ik}| = \{r + n\alpha(\alpha - \beta)\}\{r + n\alpha(\alpha - \beta) - 2a\alpha + a\beta\}$$
$$\{r + n\alpha(\alpha - \beta) - 4a\alpha + 2a\beta\} \cdots \{r - na\alpha\},$$

where the determinant is of order $n+1$ and a continuant in which the laws of the elements in the three diagonals are the following:

$$x_{i,i-1} = (n-i+2)a(\alpha-\beta),\ x_{i,i} = r-(i-1)a\beta,\ x_{i,i+1} = ia\alpha.$$

8. Professor Davis gave a complete representation of the elements of the central conic whose axes are non-similar complex quantities.

9. The object of Professor Cajori's paper was to collect all available data bearing on the invention of the slide rule and to decide between the rival claims of Gunter, Wingate, and Oughtred. He concluded from the evidence that the slide rule was invented by William Oughtred, agreeing in this with Augustus DeMorgan although most writers of the present time attribute the invention to Edmund Wingate. Much of the literature bearing on the subject and consulted by Professor Cajori was not accessible to DeMorgan. The rest of the papers were read by title.

A very pleasant feature in connection with Section A was a smoker held on Tuesday evening, which brought the members of the section into closer personal contact with one another. In a general account of a meeting of the British Association the social and semi-popular scientific functions must not be forgotten, for these have their distinct values in the purposes of the organization. There were several well attended semi-popular scientific lectures in the evenings and quite a number of garden parties and receptions, both private and official, were given in honor of the members of the Association.

At the conclusion of the meeting the officials of the Association together with a number of invited guests, in all a party of nearly two hundred, made an excursion to the Pacific coast over the main line of the Canadian Pacific Railway. Short stops were made at the principal towns en route and at a number of the mountain resorts, and Victoria was reached by boat from Vancouver. The return trip was made by way of Edmonton, from which city the special train was run over the Canadian Northern Railway back to Winnipeg.

The meeting of the British Association next year will be held in Sheffield, England.

J. C. FIELDS.

THE SALZBURG MEETING OF THE DEUTSCHE MATHEMATIKER VEREINIGUNG.*

THE twentieth annual meeting of the Deutsche Mathematiker-Vereinigung was held at Salzburg, Austria, September 19–25, 1909, in affiliation with the eighty-first convention of the Society of German naturalists and physicians.

An informal reception was held in the Kurhaus on the evening of September 19, and the first general session of all the sections and affiliated societies took place in the Aula the following morning.

The five sessions of the Vereinigung were held at the Realschule. The following papers were read:

(1) F. ENGEL, Greifswald, "Hermann Grassmann."

(2) E. WAELSCH, Brünn, "Applications of the theory of invariants of binary forms."

(3) G. PICK, Prague, "Differential equations of the periods of hyperelliptic functions."

(4) Miss E. NOETHER, Erlangen, "On the theory of invariants of forms of n variables."

(5) G. KOHN, Vienna, "On a group of theorems of projective geometry."

(6) R. ROTHE, Clausthal, "On the theory of isothermal surfaces."

(7) R. MÜLLER, Darmstadt, "On the instantaneous motion of similarly varying plane systems."

(8) R. MEHMKE, Stuttgart, "Contributions to the kinematics of rigid spatial systems, and those varying in affinity."

(9) M. GRÜBLER, Dresden, "The criterion for constrained motion of a system of screws."

(10) E. SALKOWSKI, Charlottenburg, "On a remarkable class of deformation surfaces of hyperboloids of revolution."

(11) H. WIENER, Darmstadt, "Application of kinematic considerations to the construction of curves."

(12) E. STÜBLER, Stuttgart, "The systems of accelerations in the motion of a rigid body."

(13) H. WIENER, Darmstadt, "On some new kinematic and geometric models."

* Translated from Dr. Dintzl's manuscript by Professor VIRGIL SNYDER.

(14) R. SKUTSCH, Dortmund, "Explanation of some demonstration apparatus."

(15) H. JUNG, Hamburg, "Report on the theory of algebraic functions of two variables."

(16) S. GÜNTHER, Munich, "Mathematical and physical geography of Leonhard Euler."

(17) E. HOPPE, Hamburg, "The sexagesimal system and the division of the circle of the Babylonians."

(18) L. G. DU PASQUIER, Zürich, "Concerning the question of the most suitable base for a system of numbers."

(19) W. VELTON, Kreuznach, "The development of elliptic functions."

(20) H. HAHN, Vienna, "Report on the theory of linear integral equations."

(21) A. KORN, Munich, "Solution of the linear integral equation by the method of successive approximations."

(22) W. WIRTINGER, Vienna, "Concerning conformal representation by means of abelian integrals."

(23) O. PERRON, Munich, "On the behavior of the integrals of linear difference equations at infinity."

(24) L. G. DU PASQUIER, Zürich, "On integral tettarions."

(25) E. TIMERDING, Strassburg, "Kinematic models."

(26) R. MEHMKE, Stuttgart, "The calculating machine 'Euclid.'"

(27) E. PAPPERITZ, Freiberg, "The kinodiaphragmatic projection, a new method of geometric representation."

(28) F. ENGEL, Greifswald, "On a family of curves invariant to a given differential expression."

(29) E. MÜLLER, Vienna, "Suggestions for shaping a course in descriptive geometry in technical schools and universities."

(30) E. CZUBER, Vienna, "The measure of mortality."

With the exception of numbers 3 and 6, abstracts of the papers are given below, the numbers corresponding to those in the list above.

1. Professor Engel gave a brief sketch of the mathematical development of Grassmann, which is all the more remarkable since he studied under no one. It is probably for this reason that his writings have not found more recognition, and have not exerted the influence on mathematical development that the importance and novelty of the ideas contained in them deserve.

2. Professor Waelsch generalized his definition of a spherical

harmonic in three-dimensional space, as the complete transvection of a binary form a_ξ^{2n} on the nth power of the quadratic form $(x+iy)\xi_1^2 - 2iz\xi_1\xi_2 + (x-iy)\xi_2^2$, to apply to four-dimensional space. The harmonic of R_4 appears as the bilateral complete transvection of the bilinear form $a_\xi^n b_\eta^n$ on the nth power of the form

$$(x+iy)\xi_1\eta_1 + (u-iz)\xi_1\eta_2 - (u+iz)\xi_2\eta_1 + (x-iy)\xi_2\eta_2.$$

The methods of binary invariants are not applicable to the euclidean geometry of higher spaces. The theory can therefore be regarded as a specific auxiliary for euclidean geometry of three and of four dimensions.

4. The theorems relating to the composition of forms in a ternary field are known. Their generalization to forms of n variables was impossible because no comprehensive symbolism for such forms existed. Miss Noether has developed such a symbolism by means of products of matrices. The two fundamental theorems, that concerning invariant processes and that regarding the finiteness of the system of forms, readily appear by this method.

5. The fundamental concept of Professor Kohn's paper is the "Wurf" on n elements in R_d $(n > d+2)$. These Würfe are arranged in pairs; for every one of n elements in R_d an associated Wurf of n elements in R_{n-d-2} is determined. By this means a large number of isolated theorems in projective geometry are brought into systematic relation, including Clifford's anharmonics, Rosanes's linearly independent systems of pairs of points, and Sturm's problem of projectivity.

7. Professor Müller considered first the motion through three consecutive phases, determined the center of curvature of the path curve of an arbitrary point of the system, the (1, 2) correspondence between the systems of moved points and the associated centers of curvature, and the curvature of the envelope; second, the corresponding motion through four or more such positions, introducing stationary curvature and Burmester points, as well as the locus of the pole. Finally, the determination of the phases by means of the instantaneous positions of three arbitrary points, and the introduction of branch phases.

8. Professor Mehmke discussed the torsion of an element x

of an orbit, under the most general conditions. A relation between the bisecant of the inflexional curve through x, the torsion of x and its position on the bisecant on the one hand, and the angle of contingence and the binormal component of the hyper-acceleration on the other was found, which are analogous to the formula for the normal acceleration of a plane system. The formula was generalized for certain hyperspaces, and interpretations given for the behavior at singular points.

9. The condition under which a closed system of screws consisting of n members and s screws may still have constrained motion was given by Professor Grübler as $5s - 6n + 7 = 0$, when the axes of the screws are independent. For systems of rotation the criterion is the same. If the axes of revolution intersect, giving rise to spherical motion, the criterion reduces to that of plane systems, $2s - 3n + 4 = 0$. This last relation is also true for translation.

10. The surfaces here considered appear in connection with the Bertrand curves in the following way: The rectifying cuspidal edges of two associated Bertrand curves have the same rectifying surface. After proving this theorem Professor Salkowski considered various special cases. The curves which give rise to cylinders are obtained by regarding a plane as the developable of an asteroid. The latter should now be twisted, keeping the curvature invariant, into a helix. The axes of the curve will become Bertrand curves and the altitudes of the triangles formed by the axes and a tangent will define two deformation surfaces of a hyperboloid of revolution.

11. When a point describes a plane curve the curvature is determined by the velocity and acceleration. Professor Wiener showed by means of numerous examples how to construct the tangent and radius of curvature, when the law of motion is properly chosen. If the circle of osculation be replaced by the parabola of osculation, these constructions can be simplified, and generalized to properties of contact of any order.

12. Professor Stübler investigated the instantaneous motion of a rigid system defined by the vector of the velocity of translation v, the rotation vector n, and the position of the axis of the screw. He showed how to construct the vector defining the acceleration, or conversely, when this is given, to find the

preceding elements. Then followed a construction for the axis of curvature, and the equation for the curve of inflexions, obtained as the locus of the centers of all possible accelerations compatible with the system. The case in which the velocity of rotation is perpendicular to the vector was studied in detail.

13. Professor Wiener exhibited two groups of models, invented by the author, eight to illustrate linkage quadrilaterals, and thirteen to illustrate linkage surfaces. The necessary and sufficient condition that a ruled surface be such a surface with regard to its generators and a system of directrix curves was given. A planigraph constructed on these principles was capable of drawing a complete circle of 64 cm. with an apparatus 40 cm. high.

14. Dr. Skutsch exhibited an apparatus for demonstrating the theorem of Möbius regarding the equilibrium of varying systems, keeping similarity as an invariant property.

15. In the report of Professor Jung, the investigation was restricted to linear systems of curves and the various invariant numbers that appear in functions of two independent variables.

16. It was shown by Professor Günther that the productions of Euler in mathematical geography were extensive and important, including the determination of latitude and longitude, map drawing, and the shifting of the earth's axis.

17. Professor Hoppe showed that the sexagesimal system arose from the efforts of the Babylonians to determine and define directions. The fundamental angle was not a right angle, but that of the equilateral triangle. This was divided into ten equal parts, and later into 60 equal parts. The theory is confirmed by numerous writings in the cuneiform characters.

18. Dr. du Pasquier arrived at the conclusion that the most suitable basis number is 4.

19. Starting from the differential equation defining the logarithm of an elliptic function, Dr. Velten derived the expansion in series of the Jacobi function.

20. On account of the limited time available for preparation Dr. Hahn confined his report to the Fredholm equation. He considered the solution by means of the Neumann series, the

derivation of the equation from an algebraic one by a limiting process, the memoirs of Plemelj and Goursat on the unsymmetric kernel, those of Hilbert and Schmidt on the symmetric kernel, and finally the method of Hilbert of an infinite number of variables.

21. The paper of Dr. Korn is an extension of that in the *Comptes Rendus* of June 24, 1907, by means of the methods of Poincaré, published in the *Palermo Rendiconti* in 1904, analogous to that employed in the solution of the differential equation $\Delta\phi + \kappa^2\phi = f$. Results already found by Hilbert, Fredholm, and others, were rediscovered, and an explanation of an infinite kernel of degree $\lambda < 1$ was given.

22. Professor Wirtinger first showed that every Riemann surface of an abelian integral can be uniquely obtained by the repetition of a finite number of convex rectilinear polygons. The process was carried through for normal elliptic integrals of the second kind, and for hyperelliptic integrals of the first kind. In the second case the dissection of the surface may not be unique, so that each sheet may be mapped upon a plane hexagon having given properties. From this standpoint it follows immediately that the transformation group of the periods can for $p = 2$ be generated by two operators.

23. When in the difference equation of order r

$$D_{\nu+r} + a_1^{(\nu)} D_{\nu+r-1} + a_2^{(\nu)} D_{\nu+r-2} + \cdots + a_r^{(\nu)} D_\nu = 0$$
$$(a_r^{(\nu)} \not\equiv 0) \quad (\nu = 0, 1, 2, \cdots)$$

the coefficients $a_i^{(\nu)}$ approach a power of ν, say $a_i^{(\nu)} \doteq \nu^{\kappa_i}$, then the integrals approach $\nu!$. The exponents of $\nu!$ are functions of κ only, and can be immediately found by a simple geometric construction. Dr. Perron discussed equations which are related to the above form, or can be generalized from it, and applied his results to linear differential equations.

24. The tettarions discussed by Dr. du Pasquier are defined as a special class of complex numbers that are related to linear substitutions. The idea of the field was developed and some fundamental theorems established regarding tettarion integers.

25. Professor Timerding exhibited his model, recently manufactured by Schilling, to illustrate systems varying in affinity.

In them the Nurnberger rails are applied to construct triangles and tetrahedra.

26. Professor Mehmke mentioned, as two principal advantages of the calculating machine in question, the construction of the gear and the automatic division.

27. The procedure explained by Professor Papperitz and illustrated by the lantern consists essentially in the varying role of rapidly moving models which serve to generate a picture, to translate it, and to develop it. For example, consider two diaphragms with different width of slit, in rapid rotation. The intersections of the strips of light will generate a variety of curves. On the other hand, by means of a fixed slit, a model can be projected which will give rise to as great a variety of space curves on the changing surface of the model. Curves can also be generated by appropriate shadows.

28. The calculus of variations associates with every differential expression $w(x, y, y')dx$ a covariant system of curves, the extremals, for which the first variation of the integral $\int wdx$ vanishes. Professor Engel points out that the extremals are only the first links of an endless chain of systems of curves of this kind. If ∞^2 curves c be defined by $y'' = \phi(x, y, y')$ and $\int wdx$ defines the length of arc along a curve c, and if two particular curves of the system are fixed by having given tangents at a given point, and finally if ds_0 is the difference of their length of arc at the point of intersection, then a relation $\psi(x_0, y_0, y_0', dx_0, dy_0, dx_0', dy_0', ds_0) = 0$ exists. If this relation be linear and homogeneous in the differentials, it is an extremal. If it be homogeneous and of degree n, a category of covariant curves results, and indeed the given ϕ defined by a partial differential equation. The concept can be generalized in various directions.

29. Professor Müller emphasized that descriptive geometry should be taught in the technical schools from the standpoint of its practical applications, and that elementary theoretical instruction in it should be provided in all secondary schools. The speaker then explained his own method, and illustrated it with over 300 drawings. He further urged that all candidates for positions as teachers should not only hear courses in descrip-

tive geometry, but also take part in extensive constructive exercises.

30. Professor Czuber first pointed out that from the result of the investigations of the statistical reports furnished by 60 English and by 28 Austrian insurance companies it is clear that the probability of death is not only a function of the age, but also of the length of the term of insurance. Moreover, other factors must be considered, such as sex, occupation, and personal equations in medical certificates. The problem of the measure of mortality is not so much to determine the functional relation between these numbers, which now seems impossible, as to determine individual cases empirically. The details of this procedure were then illustrated by some numerical examples.

In the general sessions, mention should be made of the paper by Professor Einstein, Bern, "The recent changes which our views of the nature of light have undergone."

In the business meeting of the Vereinigung, which was held on Thursday, September 23, reports of the various committees and officers were read, and appointments for the following year were made. Professors Krause and Schoenflies retired from the executive committee, and their places were filled by the election of Professors E. Czuber and R. Müller. The library and bibliography committee was continued. Professor Rudio reported on the status of the publication of the works of Euler.

Ample provisions were made for social intercourse, two or more entertainments being held every evening. All the participants felt that it was a most successful occasion. The next meeting will be held in Königsberg.

E. DINTZL.

GERGONNE'S PILE PROBLEM.

BY DR. H. ONNEN, SR.

IN volume I (1895), page 184, of the BULLETIN, Professor L. E. Dickson has treated Dr. C. T. Hudson's solution of the problem:* To deal a pack of ab cards into a piles of b cards each and so stack the piles after each deal that after the nth deal any selected card may be the rth in the whole pack.

* *Educational Times* Reprints, 1868, vol. 9, pp. 89-91. I have tried in vain to lay my hand on Dr. Hudson's article.

Professor Dickson gives the following result. Let $p_1, p_2, \cdots, p_n$ be the places the pile of the selected card is to hold after the first, second, $\cdots$, nth stacking of the piles; then these numbers will be the successive remainders on dividing n times by any integer a lying between the limits

$$\frac{a^{n-1}r - b}{b} + \frac{a^n - 1}{a - 1} \quad \text{and} \quad \frac{a^{n-1}(r-1)}{b} + \frac{a^n - 1}{a - 1}$$

or being equal to one of them.

Moreover Professor Dickson points out that in the particular case

$$p_1 = p_2 = \cdots = p_n = \frac{a+1}{2} \quad \text{and} \quad r = \frac{ab+1}{2}$$

n stackings are necessary and sufficient, where n is the least integer for which $a^{n-1} \geqq b$, and so proves the incorrectness of Dr. Hudson's condition for that case, viz., $a^{n+1} + 2 \geqq b$.

As to the general problem the number n of stackings that are necessary and sufficient may be computed in the following manner:

Since the difference v between the two limits is

$$v = \frac{a^{n-1}}{b} - 1,$$

the number of stackings n must be at least so great as to make

$$a^{n-1} \geqq b.$$

The least value of all is consequently that which satisfies

$$a^{n-1} \geqq b > a^{n-2}.$$

But there does not always exist an integral value for n between the two stated limits inclusive.

Suppose the lower limit

$$\frac{a^{n-1}(r-1)}{b} + \frac{a^n - 1}{a - 1} = \delta + \frac{\rho}{b},$$

δ being an integer and $\rho < b$. Then the upper limit will be

$$\delta + \frac{\rho + a^{n-1} - b}{b},$$

and there will be no integer between these limits inclusive if

$$\rho > 0 \quad \text{and} \quad \rho + a^{n-1} - b < b$$

or if

$$0 < \rho < 2b - a^{n-1}.$$

But, $(a^n - 1)/(a - 1)$ being always an integer, ρ is the remainder on dividing $a^{n-1}(r - 1)$ by b.

Hence we may lay down the following rule for computing the smallest number of stackings that may bring the selected card on the rth place in the whole pack:

Take n so that

$$a^{n-1} \geqq b > a^{n-2}.$$

Divide $a^{n-1}(r - 1)$ by b. If the remainder ρ be zero or $\geqq 2b - a^{n-1}$, n stackings suffice. But if

$$0 < \rho < 2b - a^{n-1},$$

$n + 1$ stackings are necessary and always sufficient (since $v = a^n/b - 1 \geqq a - 1$ and a is at least 2).

In the following I propose to give a further generalization of Hudson's problem.

I suppose the number of piles formed by dealing the cards not to be at every turn the same, but say the first time A_1, the second time A_2, etc., the nth time A_n. So let the total number of cards N be

$$N = A_1 b_1 = A_2 b_2 = \cdots = A_i b_i \cdots = A_n b_n = \cdots. \qquad (1)$$

Again let $a_1, a_2, \cdots, a_i, \cdots, a_n$ denote the number of piles placed *beneath* the pile containing the selected card after the first, second, $\cdots$, ith, $\cdots$, nth stacking.

After the first deal the selected card, which for convenience sake may be called X, is among the b_1 cards of the pile containing it.

At the second deal these b_1 cards are dealt over A_2 piles, so that the pile with X now contains a smaller group of cards, among which X must needs be found. And so on. Let $x_1, x_2, \cdots, x_i, \cdots, x_n$ denote the group of cards in the pile with X after the consecutive deals, in which X must needs be found. Then $x_1 = b_1$.

After the ith deal the pile with X, counting from the bottom, consists of a group of cards, which certainly does not contain X and may be denoted by m_i; above that the group x_i, containing X to a certainty; and finally a group of cards, again certainly without X.

After the ith stacking the whole pack, counting from the bottom, consists of $a_ib_i + m_i$ cards without X, x_i cards containing X, and again a certain number of cards without X.

At the following, the $(i+1)$th, deal we have

$$a_ib_i + m_i = m_{i+1}A_{i+1} + r_{i+1}; \quad r_{i+1} + x_i = x_{i+1}A_{i+1} - s_{i+1},$$

$r_{i+1} < A_{i+1}$ and $s_{i+1} < A_{i+1}$ being the numbers of cards completing respectively the first and the last of the layers formed by the group x_i.

Putting $i = 1, 2, \cdots, n$ and taking into account that $m_1 = 0$ and $x_1 = b_1$, we get the following two series of equations

After the	*Series I.*	*Series II.*
2d deal	$a_1b_1 = m_2A_2 + r_2;$	$r_2 + b_1 = x_2A_2 - s_2,$
3d deal	$a_2b_2 + m_2 = m_3A_3 + r_3;$	$r_3 + x_2 = x_3A_3 - s_3,$
. . . . .	. . . . .	. . . . .
nth deal	$a_{n-1}b_{n-1} + m_{n-1} = m_nA_n + r_n;$	$r_n + x_{n-1} = x_nA_n - s_n,$
nth stacking	$a_nb_n + m_n = z,$	

z standing for the number of cards beneath the group x_n after the nth stacking.

Multiply the two equations representing the situation after the second deal by A_1, those after the third deal by A_1A_2, etc., those after the nth deal by $A_1A_2 \cdots A_{n-1}$, and the equation after the nth stacking by $A_1A_2 \cdots A_{n-1}A_n$. Then adding separately the equations of series I and II and for the sake of brevity putting

$$A_1 = P_1, \quad A_1A_2 = P_2, \cdots, \quad A_1A_2 \cdots A_n = P_n,$$

we get, paying attention to (1),

$$\text{from I} \qquad N\left(a_1 + \sum_{i=2}^{n} [a_iP_{i-1}]\right) = zP_n + \sum_{i=2}^{n} [r_iP_{i-1}], \qquad (z)$$

$$\text{from II} \qquad N = x_nP_n - \sum_{i=2}^{n} [(r_i + s_i)P_{i-1}]. \qquad (x)$$

If $x_n = 1$, the selected card's place in the whole pack is exactly fixed after the nth stacking.

Now the greatest number of cards with which $x_n = 1$ may be obtained amounts to $P_n = A_1A_2 \cdots A_n$, r_i and s_i being both constantly zero. In this case the equation (z) gives

$$z = a_1 + \sum_{i=2}^{n} [a_i P_{i-1}]$$

and the place of X after the nth stacking becomes

$$1 + z = 1 + a_1 + A_1 a_2 + A_1 A_2 a_3 + \cdots + (A_1 A_2 \cdots A_{n-1}) a_n.$$

This result affords the following variation of the popular trick, which, I believe, is not generally known:

Take a number of cards, being the product of several numbers, for instance $48 = 4 \times 3 \times 4$, and deal them successively into as many piles as indicated by the factors, taken in any definite succession, in our example the first time into 4, the second time into 3, and the third time again into 4 piles. After every deal the pile containing the card pitched upon is indicated, and one may stack the piles in any way one pleases, provided the number of piles beneath the pile with the chosen card be at every turn remembered. Suppose this number at the three successive stackings to be 3, 1, and 2; then the place of the selected card will be the

$$1 + 3 + 4 \times 1 + 4 \times 3 \times 2 = 32\text{nd}.$$

The calculation may be easily made during the manipulations.

Since $a_1, a_2, \cdots, a_n$ are the remainders on dividing z successively by $A_1, A_2, \cdots, A_n$, one may also easily compute the mode of stacking to be performed for shuffling the selected card to any place $(1 + z)$ fixed beforehand.

If

$$P_n > N > P_{n-1},$$

the n intended deals into $A_1, A_2, \cdots, A_n$ piles are not always sufficient to fix the place of X in the pack. This appears from the following three postulates, the correctness of which will be evident by paying close attention to the equations of the series I and II:

Postulate 1. If the quantities a_i are all zero, any x_i coming from a number $N < P_n$ can never be greater than the corresponding x_i originating in the number $N = P_n$. Hence for $a_i = 0$ any number $N < P_n$ always gives $x_n = 1$, since the number $N = P_n$ furnishes at all events $x_n = 1$.

Postulate 2. If the quantities a_i are not all zero, any x_i may be at most one unit greater than it would be if $a_i = 0$. Hence, N being $< P_n$, x_n in this case may be 1 or 2.

Postulate 3. If $x_n = 2$ and a $(n+1)$th deal and stacking are to be performed, it may happen that also $x_{n+1} = 2$. In this case we have

$$m_{n+1} = m_n, \quad r_{n+1} = s_{n+1} = A_{n+1} - 1,$$

which means that one of the two cards of which x_n consists is the last of a layer and the other the first of the next layer.

Now considering the particular case

$$A_1 = A_2 \cdots = A_n = A,$$

and consequently

$$b_1 = b_2 = \cdots = l_n = b,$$

as in Dr. Hudson's problem, the equations (z) and (x) become

$$N\sum_{i=1}^{n}[a_iA^{i-1}] = zA^n + \sum_{i=2}^{n}[r_iA^{i-1}], \qquad (z_1)$$

$$N = x_nA^n - \sum_{i=2}^{n}[(r_i + s_i)A^{i-1}]. \qquad (x_1)$$

Since the limits of r_i are $A-1$ and 0 we have

$$A^n - A \geqq \sum_{i=2}^{n}[r_iA^{i-1}] \geqq 0.$$

Carrying these limits into (z_1) we get

$$\frac{(1+z)A^n - A}{N} \geqq \sum_{i=}^{n}[a_iA^{i-1}] \geqq \frac{zA^n}{N}, \qquad (2)$$

i. e., any number

$$\sum_{i=1}^{n}[a_iA^{i-1}]$$

satisfying these conditions will give such values for $a_1, a_2, \cdots, a_n$ that after the nth stacking z cards lie beneath the group x_n containing the selected card. As however, N being $< A^n$, x_n may be 1 or 2 (postulate 2), it will sometimes be uncertain whether X be the $(1+z)$th or the $(2+z)$th card.

Putting in Dr. Hudson's limits $a = A$, $r = 1 + z$ and considering that his quantities $p_1, p_2, \cdots, p_n$ correspond to $a_1 + 1$, $a_2 + 1, \cdots, a_n + 1$ in my notation, the formula for computing $a_1, a_2, \cdots, a_n$ as proposed by Dr. Hudson may be written thus:

$$\frac{(1+z)A^n - N}{N} \geqq \sum_{i=1}^{n}[a_iA^{i-1}] \geqq \frac{zA^n}{N}. \qquad (3)$$

Now it appears on further examination that any integer

$$\sum_{i=1}^{n} [a_i A^{i-1}]$$

satisfying Dr. Hudson's formula (3) gives a mode of stacking the piles, so as to make $x_n = 1$. If however such an integer do not exist, there is yet an integer satisfying the somewhat wider limits (2) and affording a proper mode of stacking as to the value of z; but with $x_n = 2$, so that it remains doubtful whether X becomes the $(1 + z)$th or the $(2 + z)$th card, and $n + 1$ stackings are required to bring it surely to the $(1 + z)$th place.

The condition (3) may be written in this manner:

$$\frac{N \sum_{i=1}^{n} [a_i A^{i-1}]}{A^n} \geqq z \geqq \frac{N \sum_{i=1}^{n} [a_i A^{i-1}]}{A^n} - \frac{A^n - N}{A^n},$$

giving one—and only one—value of z, the mode of stacking being given, if the remainder σ on dividing

$$N \sum_{i=1}^{n} [a_i A^{i-1}]$$

by A^n be $\leqq A^n - N$; in this case $x_n = 1$. If however $\sigma > A^n - N$, no integer z can be found, x_n being 2.

Taking

$$A_1 = A_2 = \cdots = A_n = A$$

and moreover

$$a_1 = a_2 = \cdots = a_n = a,$$

so as to put the pile with X at every turn in the same place between the other piles, the equations (z) and (x) become

$$aN \frac{A^n - 1}{A - 1} = zA^n + \sum_{i=2}^{n} [r_i A^{i-1}], \qquad (z_2)$$

$$N = x_n A^n - \sum_{i=2}^{n} [(r_i + s_i) A^{i-1}], \qquad (x_2)$$

whereas

$$\frac{aN}{A^n} \cdot \frac{A^n - 1}{A - 1} \geqq z \geqq \frac{aN}{A^n} \cdot \frac{A^n - 1}{A - 1} - \frac{A^n - N}{A^n}.$$

Now an integer may be found for z if the remainder σ on dividing $aN \cdot (A^n - 1)/(A - 1)$ by A^n be $\leqq A^n - N$. If $\sigma > A^n - N$, such an integer does not exist, x_n being 2.

In the second case one may deal and stack once more, and we have to examine the question whether x_{n+1} becomes 1 or 2.

In the present case, viz., $A_i = A$ and $a_i = a$, the numbers $m_2, m_3, \cdots, m_n$ in the equations of series I are increasing till a permanent maximum is attained. This maximum amounts to

$$m_n = m_{n+1} = \frac{ab - r_{n+1}}{A-1} \quad \text{if } x_n = x_{n+1} = 1$$

or to

$$m_n = m_{n+1} = \frac{ab}{A-1} - 1 \quad \text{if } x_n = x_{n+1} = 2.$$

Since m_n and m_{n+1} are integers, ab must be a multiple of $A-1$ if $x_n = x_{n+1} = 2$. Conversely $x_n = x_{n+1} = 2$ if ab be a multiple of $A-1$, *except* if $a = 0$ or $A-1$.

Hence we have the following rule (α) for computing whether it is possible or not to fix the place of X in the pack after any number of stackings, and (β) if so, whether n or $n+1$ stackings are necessary:

(α) If $A-1 > a > 0$ and ab a multiple of $A-1$, it is impossible to fix the place of X precisely. After n or more stackings its place will be the

$$\left(ab + \frac{ab}{A-1}\right)\text{th} = \left(\frac{aN}{A-1}\right)\text{th} \quad \text{or the} \quad \left(1 + \frac{aN}{A-1}\right)\text{th}.$$

In all other cases the place of X is determined after n or $n+1$ stackings.

(β) If $a = 0$, X is the first card after n stackings. — If $a = A-1$, X is the Nth card after n stackings. — If $A-1 > a > 0$ and ab not a multiple of $A-1$, divide $aN(A^n-1)/(A-1)$ by A^n. The remainder σ being $\leqq A^n - N$, the place of X is determined after n stackings. If $\sigma > A^n - N$, $n+1$ stackings are required. In both cases X becomes the

$$\left(1 + \frac{aN-r}{A-1}\right)\text{th card} \qquad (r < A-1).$$

This number is obviously $1 +$ the integer of the quotient $aN/(A-1)$.

Example 1. Take a pack of 52 cards and make in every deal 4 piles,

$$\therefore A = 4, \quad b = 13.$$

Since

$$4^3 > 52 > 4^2,$$

$n = 3$. Neither $a = 1$ nor $a = 2$ makes ab a multiple of $A - 1$. On dividing $aN(A^n - 1)/(A - 1)$ by A^n the remainder is 4 or 8, which is $< A^n - N = 12$. After three stackings X will be:

for $a = 0$ the 1st card.
" $a = 1$ " 18th "
" $a = 2$ " 35th "
" $a = 3$ " 52d "

Example 2. Take a pack of 48 cards. Making at every turn 4 piles, we have

$$4^3 > 48 > 4^2, \quad \therefore n = 3.$$

Now $ab = 12a$ is always a multiple of $A - 1 = 3$, and for $a = 1$ or $a = 2$ it is impossible to determine the place of X exactly, whatever may be the number of stackings. After 3 or more stackings one can declare only that the selected card will be

the 16th or 17th if $a = 1$
" 32d " 33d " $a = 2$.

Example 3. Suppose $N = 231$, $A = 7$, $b = 33$, $\therefore n = 3$. $ab = 33a$ is divisible by $A - 1 = 6$ if $a = 2$ or 4. If $a = 1$, 3, or 5, the remainder σ on dividing $aN(A^n - 1)/(A - 1)$ by A^n is respectively 133, 56, and 222, the second being $< A^n - N = 112$, the two others $> A^n - N$. Hence we get

$a = 0$: after 3 or more stackings X is the 1st card.
$a = 1$: " 4 " " " X " " 39th card.
$a = 2$: " 3 " " " X " " 77th or 78th card.
$a = 3$: " 3 " " " X " " 116th card.
$a = 4$: " 3 " " " X " " 154th or 155th card.
$a = 5$: " 4 " " " X " " 193d card.
$a = 6$: " 3 " " " X " " 231st card.

If A be odd and $a = \frac{1}{2}(A - 1)$, $ab = \frac{1}{2}(A - 1)b$ will be a multiple of $A - 1$ if b be even. Hence putting after every deal the pile with X in the middle, it is impossible to deter-

mine exactly the place of X if N be even. With an odd number of cards this mode of stacking always brings X to the middle of the pack, since $1 + z = \frac{1}{2}(N + 1)$.

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THE INTEGRAL EQUATION OF THE SECOND KIND, OF VOLTERRA, WITH SINGULAR KERNEL.

BY MR. G. C. EVANS.

(Read before the American Mathematical Society, September 13, 1909.)

I.

THE integral equation of the second kind, of Volterra, is written

$$u(x) = \phi(x) + \int_a^x K(x, \xi)u(\xi)d\xi. \tag{1}$$

If the function $K(x, \xi)$ is continuous, $a \leqq \xi \leqq x \leqq b$, and the function $\phi(x)$ is continuous, $a \leqq x \leqq b$, there is one and only one continuous solution of the equation. But if $K(x, \xi)$ is not continuous in its triangular region, the case is more complicated. In I. we consider finite solutions of integral equations of which the kernel $K(x, \xi)$ is absolutely integrable, and after obtaining a theorem for that case apply it to some others where the kernel is no longer absolutely integrable. For this theorem the following conditions limit the given functions of the equation: $K(x, \xi)$ shall satisfy (A) and $\phi(x)$ shall satisfy (B).

(A) A real function of the two real variables x, ξ is to be continuous in the triangle $T: a \leqq \xi \leqq x \leqq b$, $b > a > 0$, except on a finite number of curves each composed of a finite number of continuous pieces with continuously turning tangents. Any vertical portion is to be considered a separate piece, and of such pieces there are to be merely a finite number, $x = \beta_1$, $x = \beta_2, \cdots, x = \beta_r$. On the other portions of the system of curves there are to be only a finite number of vertical tangents.

(B) In the region $t: a \leqq x \leqq b$ a real function of a single real variable x is to be continuous except at a finite number of points $\gamma_1, \gamma_2, \cdots, \gamma_s$, and is to remain finite.

Let us define the linear region t_δ, formed from t by removing the small portions $\alpha_i - \delta < x < \alpha_i + \delta$ $(i = 1, 2, \cdots, l)$; and the two dimensional region T_δ, formed from T by removing the small strips $\alpha_i - \delta < x < \alpha_i + \delta$ $(i = 1, 2, \cdots, l)$, where the δ is an arbitrarily small magnitude, and the α's, finite in number, are yet to be defined.

Under these conditions we have the

THEOREM. *There is one and only one finite solution of the integral equation* (1), *continuous in t except for a finite number of points, and these points will be among the points* $\gamma_1, \cdots, \gamma_s, \alpha_1, \cdots, \alpha_l$ (*the* α'*s to be defined below*); *provided conditions* (*A*) *and* (*B*) *and the following further conditions are fulfilled:*

(*a*) $\int_a^x |K(x, \xi)|\, d\xi$ *converges in t except for a finite number of points* $\lambda_1, \cdots, \lambda_\rho$, *and remains finite.*

(*b*) *There is a finite number of points* $\alpha_1, \cdots, \alpha_l$ [*including the points* β *of* (*A*) *and* λ *of* (*a*)] *such that when* ϵ *and* δ *are chosen at pleasure there is a length* η_δ *for which*

$$\int_y^{y+\eta_\delta} |K(x, \xi)|\, d\xi < \epsilon, \qquad (x, y) \text{ and } (x, y + \eta_\delta) \text{ in } T_\delta.$$

(*c*) *t can be divided into k parts, bounded by points* $a = a_0, a_1, \cdots, a_{k-1}, b = a_k$ *such that*

$$\int_{a_i}^{x} |K(x, \xi)|\, d\xi \leqq H < 1 \quad \begin{cases} a_i \leqq x \leqq a_{i+1}, \\ x \neq \alpha_1, \cdots, \alpha_l. \end{cases}$$

In the proof of this theorem (*a*) and (*c*) are used in showing the convergence of the expansion of the solution, and (*b*) in developing what continuity exists.

The condition (*A*) can be replaced by conditions on the integral of the kernel; for instance (*A*) and (*b*) can together be replaced by the condition which follows:

The integral

$$\int_a^x K(x, \xi) r(\xi) d\xi,$$

where $r(x)$ *is finite in t and continuous except for a finite number of points, shall converge except at most for a finite number of values of x, and the function of x thus defined shall remain finite; furthermore it shall be continuous except at most for a finite number of values of x, denoted by* $\alpha_1, \cdots, \alpha_l$, *which are independent of the choice of* $r(x)$.

A special case of this theorem has been treated by Mr. W. A. Hurwitz.* The hypotheses for this case were

(B') $\phi(x)$ is continuous in t,

(a') $\int_a^x |K(x, \xi)|\, d\xi$ converges in t,

(b') $\int_a^x |K(x, \xi)|\, d\xi$ represents a continuous function in t,

(c') $|K(x_1, \xi)| \geqq |K(x_2, \xi)|$ when $x_1 > x_2$.

Here (a') implies (a), and (b') and (c') together imply (c) and the condition just mentioned that replaces (A) and (b).

By application of the theorem of page 131, with a change of dependent variable, equations of a still more extended type may be solved. In the equations

$$u(x) = \phi(x) + \int_a^x \frac{K(x, \xi)}{f(\xi)} u(\xi) d\xi, \tag{2}$$

$$u(x) = \phi(x) + \int_a^x \frac{K(x, \xi)}{f(\xi) g(x)} u(\xi) d\xi, \tag{3}$$

and

$$u(x) = \phi(x) + \int_a^x \frac{K(x, \xi)}{f(\xi) g(x) \prod_{i=1}^{p} \{[\xi - \psi_i(x)]^{\lambda_i}\}} u(\xi) d\xi, \tag{4}$$

$$\sum_{i=1}^{p} \lambda_i = \lambda < 1,$$

any one of which includes the previous ones as special cases, $K(x, \xi)$ shall satisfy (A) and be finite, $\phi(x)$ shall satisfy (B), $f(x)$ and $g(x)$ shall be continuous in t and unequal to zero except at the point a where they may vanish in any way, and the various ψ's shall be continuous functions of x. Then,

In (2), if $\phi(x) e^{\int_x^b \frac{dx}{|f(x)|}}$ remains finite as x approaches a, there is a solution that vanishes at a as sharply as const. $e^{-\int_x^b \frac{dx}{|f(x)|}}$.

In (3), if $\phi(x) g(x) e^{\int_x^b \frac{dx}{|f(x) g(x)|}}$ remains finite as x approaches a, there is a solution that vanishes at a as sharply as

* As a problem in Professor Bôcher's course in Integral Equations, Harvard University, in 1907–1908.

$$\text{const.}\ \frac{1}{g(x)}\, e^{-\int_x^b \frac{dx}{|f(x)g(x)|}}.$$

In (4), if $\phi(x)g(x)e^{\alpha\int_x^b \frac{dx}{[\,|f(x)g(x)|\,]^{(\nu+\lambda)/\nu}}}$ remains finite as x approaches a, where α is a certain constant, and ν any constant such that $1-\lambda>\nu>0$, there is a solution that vanishes at a as sharply as

$$\text{const.}\ \frac{1}{g(x)}\, e^{-\alpha\int_x^b \frac{dx}{[\,|f(x)g(x)|\,]^{(\nu+\lambda)/\nu}}}.$$

There may however be more than one finite solution of these equations.

II.

In this section we consider kernels that are not absolutely integrable. We have the following introductory theorem:

Let the kernel of the integral equation (1) be in the form

$$\frac{K(x,\,\xi)}{G(x,\,\xi)},$$

where

$G(x, \xi)$ is analytic in T;*

$K(x, \xi)$ is continuous in T, and $\phi(x)$ continuous in t;

$K(x, \xi)$ vanishes at most at a finite number of points in T at which $G(x, \xi)$ also vanishes.

Then there is no solution of (1), continuous in t except for a finite number of points and not identically vanishing through any subinterval of t, unless the kernel $K(x, \xi)/G(x, \xi)$ can be written in the form

$$\frac{\overline{K}(x,\,\xi)}{g(x)f(\xi)},$$

where $\overline{K}(x, \xi)$ is continuous in T, and $f(x)$ and $g(x)$ are analytic in t.

If $K(x, \xi)/G(x, \xi)$ cannot be rewritten as $\overline{K}(x, \xi)/g(x)f(x)$ for values of ξ, $x_1<\xi<x_1'$, $x_2<\xi<x_2'$, $\cdots$, $x_p<\xi<x_p'$, it is necessary for all such values of ξ, if the integral is to converge, that $u(\xi)=0$. Hence, in general, under such conditions, there

* If we replace the triangle T by the square $S: a\leqq x\leqq b$, $a\leqq\xi\leqq b$, this theorem holds also for the equation with constant limits

$$u(x)=\phi(x)+\int_a^b \frac{K(x,\,\xi)}{G(x,\,\xi)}\,u(\xi)d\xi.$$

will be no solution of the integral equation (1). For that there be a solution under such conditions it is necessary, as is obvious from the form of the equation (1), that the given function $\phi(x)$ satisfy the equations

$$\phi(x) = \int_a^{x_j} \frac{K(x, \xi)}{G(x, \xi)} u(\xi) d\xi, \quad x_j < x < x_j' \quad (j = 1, 2, \cdots, p);$$

wherefore the $\phi(x)$ cannot be chosen arbitrarily in those subintervals of t. The solution when it exists is independent of the value of the kernel in the strips for which $x_j < x < x_j'$, or $x_j < \xi < x_j'$ $(j = 1, 2, \cdots, p)$.

This prepares us to state the

THEOREM. *Let the kernel of* (1) *be in the form*

$$\frac{K(x, \xi)}{f(\xi)g(x)},$$

where

1° (*a*) $K(x, \xi)$ *is continuous in* T, *and* $f(x)$, $g(x)$ *and their first derivatives are continuous in* t;

(*b*) $\partial K(x, \xi)/\partial x$ *satisfies* (*A*), *page 130, and is finite in* T;

(*c*) $\phi(x)$ *is continuous in* t *except at* $x = a$ *and is such that the function* $\phi(x)g(x)$ *and its first derivative satisfy* (*B*), *page 130.*

2° *The function* $f(x)g(x)$ *is greater than zero in the neighborhood of* a, *and at* a *vanishes in such a way that* $\int_a^x \frac{dx}{f(x)g(x)}$ *is not convergent;*

3° $\lim_{x=a} [K(x, x) - K(a, a)]/(x - a)^\nu$ *exists, where* $K(a, a) \neq 0$, *and where* ν *is some number lying between* 0 *and* 1 $(1 > \nu > 0)$ *and is also greater than* $1 - 1/\{d[f(x)g(x)]/dx\}_{x=a}$;

4° $\lim_{x=a} \phi(x)g(x) = 0$.

Then, under the foregoing conditions,

(*i*) *if* $K(a, a) < 0$, *there exists one solution of* (1) *continuous in the neighborhood of* a *and at* a, *and*

(*ii*) *if* $K(a, a) > 0$, *there exists a one-parameter family of solutions of* (1) *continuous in the neighborhood of* a *except perhaps at* a *itself. As* x *approaches* a, *each solution remains less in absolute value than some constant times* $f(x)/(x - a)^\nu$.

If $K(a, a) < 0$ we may take $\nu = 0$ without change in the

theorem. Also if $K(a, a) > 0$ and $[df(x)g(x)/dx]_{x=a} < 1$, we may take $\nu = 0$.

A slightly more special theorem, equivalent to taking $\nu = 1$, is obtained by inserting in 1° : *$\partial K(x, \xi)/\partial\xi$ satisfies (A) and is finite in T,* and dropping all of 3° except $K(a, a) \neq 0$.

If we write $K(x, \xi) = K(a, a) + \lambda[K(x, \xi) - K(a, a)]$, the solutions specified in the theorem of page 134 are analytic in the parameter λ, and are the only solutions continuous in the neighborhood of a, except possibly at a, that are analytic in λ. They are also the only solutions continuous in the neighborhood of a, except possibly at a, that satisfy the conditions

$$(a) \qquad \lim_{x=a} \Phi(x) = 0,$$

$$(b) \qquad \Phi'(x) \text{ remains finite,}$$

where

$$\Phi(x) = \int_a^x \frac{K(x, \xi) - K(a, a)}{f(\xi)}\, u(\xi)d\xi.$$

There are no solutions that satisfy these conditions if 1°, 2°, and 3° of page 134 hold, but not 4°.

III.

So far we have considered only finite solutions, or at most solutions that become infinite at $x = a$ to an order not greater than the first. It is possible, however, to limit the totality of solutions as to character.

THEOREM. *Let the kernel of* (1) *be in the form $K(x, \xi)/f(\xi)g(x)$, where*

1° (*a*) *$K(x, \xi)$ is continuous in T,* and $f(x)$, $g(x)$ and their first derivatives are continuous in t;*

(*b*) *$\partial K(x, \xi)/\partial x$ and $\partial K(x, \xi)/\partial\xi$ satisfy A, and are finite in T;*

(*c*) *$\phi(x)$ is continuous in t except perhaps at a, and is such that the function $\phi(x)g(x)$ and its first derivative satisfy (B);*

2° *The function $f(x)g(x)$ vanishes at most a finite number of times in t;*

3° *On any horizontal line $\xi = \xi_0$ cutting T there is at least one point in T for which $K(x, \xi) \neq 0$.*

* If we replace T by S this theorem holds for the equation with constant limits.

Then all the solutions of (1) *continuous in* t *except for a finite number of points are such that the function* $u(x)g(x)$ *remains continuous in* t.

This theorem has several applications. If neither $f(x)$ nor $g(x)$ vanishes in t, there can be no solution becoming infinite at any point in t; therefore the continuous solution is the only solution of the equation continuous except at a finite number of points. If $K(a, a) \neq 0$, and if $\lim_{x=a} \phi(x)g(x) = 0$, the theorem of page 134 holds, as we have already noticed. The solutions there given are the only ones continuous except at a finite number of points, such that $d[u(x)g(x)]/dx$ remains finite in t; they are also the only solutions possible, continuous except at a finite number of points, provided that $K(x, \xi) - K(a, a)$ vanishes identically when $\xi = x$.

If the kernel of the integral equation (1)* is analytic in T, and if $\phi(x)$ is continuous in t, a proof similar to that of the above theorem shows that the continuous solution is the only solution of (1) continuous in t except for a finite number of points.

HARVARD UNIVERSITY,
September, 1909.

DESCRIPTIVE GEOMETRY.

Lehrbuch der darstellenden Geometrie für technische Hochschulen, Volume I. By Professor EMIL MÜLLER, of the Imperial Technical School at Vienna. Leipzig and Berlin, Teubner, 1908. xiv + 368 pages, 273 figures, and three plates.

Vorlesungen über darstellende Geometrie. By GINO LORIA. Volume I: *die Darstellungsmethoden.* Authorized German translation from the Italian manuscript, by FRITZ SCHÜTTE. Teubner's Sammlung, volume XXV_1. Leipzig and Berlin, Teubner, 1907. xi + 218 pages and 163 figures.

Descriptive Geometry, a treatise from a mathematical standpoint, together with a collection of exercises and practical applications. By VICTOR T. WILSON, Professor of drawing and design in the Michigan Agricultural College. New York, John Wiley and Sons, 1909. 8vo, viii + 237 pages and 149 figures.

* If the kernel of the integral equation with constant coefficients is analytic in S and if $\phi(x)$ is continuous in t, the continuous solutions are the only solutions continuous except for a finite number of points.

DURING the last few years Teubner has published twenty-four treatises, several of them of two or more volumes, on descriptive geometry, and his latest circulars announce that ten more are in an advanced state of preparation. And this is only one instance; a number of other publishers announce the early appearance of one or more books on the same subject, and the periodic literature of Europe (particularly German and Italian) contains during the last ten years some five hundred memoirs, essays, notes, solutions, etc., pertaining to this science. Under these circumstances the question may be asked, why still another book in a language represented by so many? But an examination of Professor Müller's work will dispel any doubts as to the wisdom of its publication. Descriptive geometry was taught to most of us in America who had any instruction at all in the subject, not as a science, but rather as a clever device for producing certain graphical representations. Courses in it are given only in our technical schools, conducted by engineers for purely practical purposes, and largely without proofs. The students soon think of the procedure as empirical and frequently wonder why the drawings come out so well, when they do not really know just what they are doing. For this reason, many of our American graduates who have extensive responsibilities in draughting operations have had to learn most of the elaborate processes in the offices, and begin years later really to understand the geometric principles upon which the constructions were based.

The present book has a very different character; a student who masters it will not doubt that the various devices will accomplish the desired ends. The author has taught the subject for years, has had extensive experience with graphical methods, and by his achievements in other fields of mathematics has proved that he can speak with authority. While on every page emphasis is laid on the fact that the entire subject is to be regarded as an auxiliary science for engineers and architects, and hence the practical applicability is the principal aim, yet nothing is taken for granted, every step being carefully analyzed. One striking feature is the early insistence on the interpretation of a figure as a whole, rather than simply the sum of a number of individual points and lines. This is an excellent means for retaining the active interest of the student. Thus, a general parallel projection of a hexagonal pyramid is discussed as early as page 25, and shades and shadows are systematically intro-

duced on page 48. Within the next few pages the ground, front, and end elevations of complicated frames are constructed, and their shadows found.

A concise system of abbreviations is used throughout. This greatly aids in reading demonstrations, the only objection being that it is different from that of other writers on this and related subjects.

In the first part of the book, which treats of the ordinary properties of rectilinear figures, two principles are successfully employed, both of which are unusual at this stage, but are always met with in practice. The first is the frequent use of the general profile, that is, projection on a plane perpendicular to but one of the fundamental planes; the other is the early omission of the ground line. These principles are used throughout the volume.

Another feature is the prominence of the discussion of projective properties, which always precede the metrical ones. Incidentally, the student is gradually gaining a comprehensive knowledge of projective geometry, but always in organic relation to graphical problems. Affinity is indeed given a separate consideration, the first part of which is quite independent of graphical notions.

The second part of the book is devoted to the general theory of curves and surfaces. The treatment is partly algebraic and partly differential. Here the author seems rather too ambitious; he acknowledges in the preface that the presentation may be too brief, but hopes by presupposing considerable knowledge of analytics and the calculus that the student may see his way through. In thirty pages we find such varied concepts as Plücker's numbers for algebraic plane curves, the circular points and absolute circle, the imaginary generators of ellipsoids, and a number of properties of space curves, including lines of curvature, asymptotic, and geodesic lines on a given surface. Either the student must have learned these things before, or he will not sufficiently know them even after this discussion. This is the only part of the book that does not seem to be carried out in the best way.

On the other hand, the applications of these principles to the curves and surfaces of the second order, which is found in the succeeding chapters, is exceptionally well done. A large number of theorems usually found in books on analytics are established, including a direct graphical determination of the center

of curvature of an arbitrary point on an ellipse, by means of three orthogonal projections. Cones, cylinders, and developables are considered before the general quadric is taken up. Much of this matter is far more extensive than is usual in books on descriptive geometry; it compares roughly with the corresponding chapters of Reye's Geometrie der Lage.

The excellent chapters on surfaces of revolution and helicoids contain a rich fund of information, which is of value for purposes other than graphical representation. The theory of illumination is developed, curves of equal illumination determined, and the results compared with true and apparent contour.

Among the commendable details of the book we may mention the extensive index, the excellent figures, the free-hand lettering, and the frequent instructions and admonitions as to procedure, including the preparation of washes, tints, inks, etc. — making it a serviceable hand-book as well as a thorough theoretical treatise.

The above will indicate why the publisher should undertake the production of another book on descriptive geometry. A second volume is to consider central perspective, axonometry, and related subjects.

The present first volume of Professor Loria's treatise is concerned entirely with methods of graphically representing configurations composed of points, straight lines, and planes. It presupposes some knowledge of projective geometry of the plane, including self-corresponding and double elements, Pascal's theorem, Desargues's theorem, and poles and polars. The work is divided into five books; the first (88 pages) discusses the ordinary problems of descriptive geometry, but the proofs are unusually systematic, and all particular and exceptional cases are treated. The constructions are all reduced to depend upon four fundamental ones. Results are expressed in bold-faced type and a number of unsolved exercises follow each article.

The profile and certain oblique planes are frequently employed, and frequent use is made of rotation. Among the themes treated in this short space are the determination of the two transversals of four skew lines and the construction of the regulus defined by three lines.

The second book considers most of the same problems from

the standpoint of central perspective. Given a center C and a plane π. Let a line pierce π in T. Through C draw a line parallel to the given one, cutting π in I. The projection of the line is then defined by TI. Similarly, a plane is expressed in terms of its trace t and the parallel line i. The third book has for subject the projective equivalent of the preceding one, the point C now being at infinity in the direction normal to π. The other defining element is a plane parallel to π at a known distance from it. The chapter closes with an instructive comparison of central and double orthogonal projection.

The discussion of axonometry in the fourth book (45 pages) is an excellent theoretic presentation, but rather too brief to be of greatest use. While all the necessary steps are given, they are so brief, and treated each alone, that the guidance of a teacher is necessary to the average reader. In the preceding parts one could easily read the text and solve the assigned problems without such assistance. The chapter on orthogonal axonometry is followed by one on oblique parallel, and one on central, but these are hardly more than outlines. Finally, there is added as a fifth book a brief treatment of photogrammetry, *i. e.*, the science of constructing a third perspective of a space figure when two are completely given, or of finding either a central or orthogonal projection when several are partially given, e. g., the drawings given, but the position of the corresponding centers not specified. The discussion of these problems requires many more theorems from projective geometry, the proofs of which are supplied. The results are very interesting; while considerable knowledge and higher maturity are required on the part of the student, this discussion cannot help being a valuable incentive to the more advanced student.

The book treats the problems it proposes with thoroughness and skill, but a reader will after all probably not get a proper idea of the beauty or of the usefulness of descriptive geometry by reading it, because it treats them in an isolated matter, and does not develop the most useful power of being able to interpret a complicated drawing as a whole. The figures of a few building fronts or plans treated entirely in straight lines would add greatly to the usefulness of this useful book. Probably such matters will be adequately treated in the second volume.

In the preface of Professor Wilson's book we find the following statement: "Descriptive geometry is essentially a math-

ematical subject. The applications of its principles to the making of working drawings, however, and the modifications which are made to meet the contingencies of practice have had a tendency to obscure this fact and like other theoretical subjects it has suffered mutilation in the interest of short cuts to immediate practical uses. But does not technical education after all consist chiefly in an equipment of sound theory?" In the treatment of the configurations consisting of straight lines and planes the author has well justified this statement, and has made a real improvement on the presentation found in most American texts. The presentation is didactic, the explanations being full, and theorems are followed by a number of numerical exercises. The idea of rotation of one or both axes is brought in on page 17 and extensively employed in subsequent problems.

The discussion of curves and surfaces is much less satisfactory, and only partly fulfils the promise in the second part of the title. Thus, in defining parallel lines (page 2) we meet the statement: "Hence parallel lines are said to have two points in common at infinity"; on page 89 we find: "Double curved surfaces are generated only by the motion of curves and have no straight line elements." The words consecutive and coincident on page 93 are confusing; the rectification of a curve, defined on page 95, is obtained by rolling the curve out on its tangent. The word touch, as applied to space curves, is everywhere incorrectly used. According to the definition of a warped surface on page 139 no ruled surface can have a double curve. A number of theorems are worded much too broadly, as they apply only to particular cases; thus, on page 149, after discussing a method for drawing a tangent plane at a point on a general quadric, we are assured that it is applicable to any surface having two systems of rectilinear generators. The theorem on page 151 refers only to rectangular hyperboloids. Unfortunately, the book is full of similar statements; many of them are found in most of the American text-books on the subject, but a few are new contributions. However, the author should not be too severely criticized; he has at least felt the need of more light, and has made decided improvements in the early part of the book.

In view of the sharp contrast between this, one of the best American text-books on descriptive geometry, and those mentioned above we must conclude that there is still room for a good treatise on the subject in English.

VIRGIL SNYDER.

ELEMENTARY MECHANICS.

A First Statics. By C. S. JACKSON, M.A., R. M. Academy, Woolwich; and R. M. MILNE, M.A., R. N. College, Dartmouth. [Dent's Mathematical and Scientific Text-books, edited by W. J. Greenstreet.] London, J. M. Dent & Co., 1907. Crown 8vo. vii + 380 pp. 4 shillings.

A Text-book of Mechanics. By LOUIS A. MARTIN, JR., M.E., A.M., Stevens Institute of Technology. Vol. I, *Statics*, 1906. 12mo. xii + 142 pp. $1.25. Vol. II, *Kinematics and Kinetics*, 1907. 12mo. xiv + 214 pp. $1.50. New York, John Wiley and Sons.

PERHAPS it is immaterial to our colleges whether their average graduate knows even the simplest principles of physics, — so as to be able, for example, to explain the advantages of a simple set of pulleys, or the physical basis of time measurements, or the easy balancing of a moving bicycle, or when a leaning ladder could be safely ascended if slipping were opposed only by friction. At least, such an inference seems warranted so long as educators suppose that familiarity with records of human activities of various sorts is a compensation for ignorance of natural laws, and until they cease placing their labels of education upon persons who have not thoroughly studied even the broader sciences. The common and dense ignorance of one fundamental science could be removed by even a first course in mechanics (developed from the physical side), whether or not the average student should later elect the more mathematical study of this science, whose problems have so greatly stimulated the growth (and still defy the powers) of mathematical analysis. These two kinds of courses in mechanics are somewhat typified by the books under review.

The First Statics is not purely mathematical, as it employs numerous simple experiments; but it is far from being a laboratory manual. Experiments are suggested merely to introduce the laws — and the first ideas of mechanics should always come in that way — while deductive analysis is used throughout, though with no stress on rigor. Trigonometry is employed, but not calculus, even in finding centers of gravity.

The authors treat the lever and the equilibrium of paral-

lel forces before introducing the parallelogram of forces, which follows next with graphical methods for three concurrent forces, and leads to analytical methods of composition and resolution, and the properties of couples. Several simple machines are then explained: weighing-bridge, "penny-in-the-slot" machines, tackle, differential pulley, Chinese wheel and axle, lifting crab, worm and wheel, and screw. There is a good treatment of friction and an extended discussion of centers of gravity — of a few bodies geometrically, of some others by applying the theorems of Pappus, and of any number of particles analytically by taking moments. Graphical analysis of framed structures, the use of the link polygon, and the general analytical conditions for the equilibrium of coplanar forces are treated at some length, followed by an exposition of the relative advantages of graphical and of analytical methods, and of the usefulness of the principle of work. The volume concludes with a chapter on forces in space, and a good set of miscellaneous problems.

Features of this very worthy text which deserve special commendation are the presence of occasional questions as to the probable extent and cause of error; the carefully graded treatment, with simplest beginnings; the excellent set of some 700 problems (especially good on taking moments); and the admirable treatment of the parallelogram of forces, not offering a pretended proof of it, but assuming it as the result of experience, after the student has been convinced by experiments. Indeed, important throughout is the inductive presentation of new ideas, whereby the student is prepared by experiment or special example for the statement of a principle. Of course, in the teaching of pure mathematics, this office of the experiment is not dispensed with, but filled by the numerical example. To illustrate: the average student of analytical geometry is puzzled if directly confronted with $S_1 + kS_2 = 0$ as the equation of a straight line through the intersection of the lines $S_1 = 0$ and $S_2 = 0$. But if he has first seen several numerical cases, and has observed how the new line changes for different values of the parameter, he can readily grasp the facts concerning the general equation, which should not even be written until the examples have been understood. Never should any new concept be introduced save through special cases. The soundness of this pedagogic principle is so incontestible that one marvels at the small part it plays in our texts.

But to return: there are parts of the First Statics which are open to criticism. In proving the theorems of Pappus for curves, would it not be better to employ exact (though elementary) limit statements than to say (page 224): "since the proposition holds for the sum of all the rectangles, it will hold for the area of the curve itself?" Or (page 228): "the solid figure formed . . . will closely coincide with the sphere. We therefore infer that the surface of the sphere itself is equal to $4\pi R^2$." It might be well in some proofs to point out what statements are not based on earlier principles but are merely assumed as true; thus it would be interesting to question whether step (1) in Stevin's solution (page 62, example 5) need be admitted unless perpetual motion is supposed impossible. It is somewhat careless to equate a pure number to a number of dimensional units, as (page 238) "$225/80 = 2.81$ feet;" or (page 302) "$125 \cos 40°/\sin 65° = 106$ lbs. nearly." * And such experiments as that on page 75 seem too clumsy to be useful.

The following misprints have been noted: page 76, example 33, for 30 read 31; page 114, top, for orignial read original; page 123, bottom, for enses read senses; page 187, bottom, for revolving read resolving; page 207, figure, for interior B read B'; page 285, frame diagram, for one b read b'; page 338, line 4, for e read be. The reviewer would also suggest more paragraph headings. American students should be informed: whether (page 144) a "piece of cotton" is a thread; (page 201) that the given weight of a penny is not that of a cent; and that an English "trapezium" (page 204 et seq.) is an American "trapezoid."

Professor Martin's Text-book of Mechanics presents, on the other hand, a purely mathematical treatment, using no calculus in the first volume but introducing it gradually in the second.

The Statics begins with the parallelogram of forces, assumed as an experimental datum, and treats the composition and resolution of forces, couples, conditions for equilibrium in two dimensions, and centers of gravity (rather briefly, without the theorems of Pappus). A summary of methods is followed by the consideration of various machines: lever, wheel and axle, systems of pulleys, inclined plane and wedge, bent lever balance, differential wheel and axle, and platform scales.

* The same criticism applies to the other text under review; e. g., Vol. I, p. 2; Vol. II, p. 19.

Except for an appendix on three-dimensional structures, and problems for review, the rest of the volume is given to the graphical methods used for centroids, resultants, and framed structures.

In kinematics the composition and resolution of velocities and accelerations in the motion of a particle are followed by space-time and velocity-time curves, the composition of simple harmonic motions, and a brief treatment of the translation and rotation of rigid bodies. The author begins kinetics with some explanatory comments on Newton's laws of motion, follows with a commendable digression on the theory of dimensions; and discusses the free translation of a particle or mass center for various forces, constrained motion, and the rotation of a rigid body, moments of inertia being introduced very naturally. He then treats work and energy, and the application of the principle of work to machines; and concludes with a chapter on impact, and a good set of review problems.

Evidences that the text is the work of a teacher appear throughout both volumes; as, for example, in the lucid explanation of the angle of repose, and in the facility with which practical problems are introduced early in the first volume; or, again, in the excellent method of analyzing very fully many typical problems (although one might wish to see more of these examples used to introduce the discussion of principles). With such a text the student must, in order to solve problems, gain an understanding of the subject by study of the solved problems and general theory; while some mathematical texts give him such explicit working rules that he can mechanically follow the rules without understanding the theory, with the result that, when the memorized rule is forgotten, he has nothing left. Still worse in the reviewer's opinion is the tendency of rules to deprive the student of the necessity for formulating methods independently; for one glaring fault of our educational system is that it cultivates so little originality and ability to think. Trained men are good, but educated men are better! Working rules have their place (along with tables) in practice, but in teaching they should be used with moderation.

There is of course, no protest against collecting the results of discussions, as the author has done in several places, thereby adding further to the success of his effort (mentioned in the preface) to make the book thoroughly teachable. In fact, the reviewer believes that as a text this work ranks with the best

previous books, despite a number of criticisms which must be stated.

For one thing, the desirability of presenting at the outset all the material of the introduction, which precedes statics, may be questioned; but more serious is a logical incompleteness similar to that which pervaded our elementary geometries before the modern revision. For example, just as we used to "prove" theorems concerning the area of a circle, which had never been defined (unless by a meaningless phrase), so here (pages 36, 43), although neither the acceleration nor any component has been defined for curvilinear motion, the author proves that $\alpha = \sqrt{\alpha_x^2 + \alpha_y^2}$ and not d^2s/dt^2. Also (page 7) we find assertions concerning velocity when $\Delta s/\Delta t$ varies, although the word has been given a meaning only when that ratio is constant; and similar remarks apply to acceleration (page 11). It may easily confuse a student to read (page 38): "In curvilinear motion the velocity can even be constant and still there must be an acceleration," when his only definition of acceleration is (page 10): "the time-rate of change of velocity;" or (page 11), "acceleration is always the first derivative of velocity with respect to time."

Again from the mathematical standpoint it is unsatisfactory to have a body treated as consisting of a finite number of particles, without an adequate transition from the sign of summation to that of integration. Also unfortunate though less important is the use of three steps (page 13) to conclude that if $v = ds/dt$ and $\alpha = dv/dt$, then $\alpha = d^2s/dt^2$; and do the remarks (page 8) about t being "an equicrescent variable so that dt is constant" mean anything more than that we choose t as the independent variable and can therefore take dt constant if we wish? Will the remarks concerning a rolling wheel (page 62) be clear before the student has heard of the instantaneous center; and would it not be shorter and more natural (page 103) to let s denote the distance from the center of the earth? In the solved problem on page 18 (Volume I) the use of similar triangles could be replaced by trigonometric methods by merely lettering an angle. It is inaccurate to say (Volume I, page 46): "CA_1 can only equal CA_2 as $CA_2 \doteq \infty$;" or write as equal the alternative values for t_1 and for t_2 (Volume II, page 152). It might be well to state that F is regarded as constant (Volume II, page 71) in defining impulse, and (page 167) in defining work; and to insert (page 140, line 3) the words "to be proved" after are.

There are some typographical errors: Volume I, page 59, middle, for $\Sigma(Fd)$ read $-\Sigma(Fd)$; Volume II, page 33, omit to in sixth line from bottom; page 88, bottom, for ΣF read ΣF_x; page 101, line 2, insert a comma before e; page 165, figure, for ax and ay read a_x and a_y; page 189, line 7, for are read is.

Nevertheless, it must be said that each of the texts under review is well printed on the whole, and presents a very attractive appearance. And despite the extent of the criticisms here offered, both books should prove thoroughly practical, often suggestive and highly satisfactory. Neither text is indexed, but the deficiency is not serious in such a subject.

As these pages go to press, a second edition of Professor Martin's Mechanics is announced by the publishers. The early appearance of a second edition is perhaps the most substantial evidence of the success of the book.

F. L. GRIFFIN.

SHORTER NOTICES.

Opere Matematiche di Eugenio Beltrami. Pubblicate per cura della Facoltà di Scienze della R. Università di Roma. Milano, T. I, con ritratto e biografia dell' autore, 1902, vii + 437 pp.; T. II, 1904, 468 pp. 4to.

THESE two volumes contain 43 memoirs of Beltrami's, ranging from 1861 to 1873, and an epilogue, by Cremona which serves as an introduction. Among the papers included are those by which Beltrami earned his world-wide fame and established once for all his claim to be regarded as one of the founders of the modern subject of differential geometry. There are the memoirs on differential parameters, on the complex variable spread over an arbitrary surface, the memoirs on non-euclidean geometry, referring to ordinary and hyperspaces, the theory of geodesic lines, including the famous solution of the problem of geodesic representation of a given surface on the plane. The second volume contains also an elaborate study of the kinematics of a fluid.

If we except, perhaps, the last-named voluminous research, it may be said that the main results arrived at by Beltrami are now generally known. They have found their way into the more extensive text-books; so that we need not repeat what is, or may be, familiar to every student. Nevertheless the present

edition is highly to be welcomed. Through the necessary process of condensation an author's results often lose much of their freshness, so that the careful student will always be glad to revert to the original work. Thus the beautiful theorem concerning the deformation of an arbitrary surface to which a normal system of rays is rigidly connected comes out quite naturally where it stands (page 121 of the present edition, volume I), while the much shorter proof by which Beltrami's development has since been replaced has a somewhat artificial aspect, and fails to suggest how one may arrive at such a statement. Numerous theorems, to be found in text-books without any references, were probably discovered by Beltrami. At least Beltrami himself (who had a vast knowledge of the literature, foreign as well as Italian, and was most careful in bestowing due credit on others) apparently considered them as new. And there are many other interesting theorems still as fresh and new as they were when published for the first time.

A peculiar charm of these writings, not often to be found in work of high originality, lies in the simplicity of Beltrami's exposition. He was evidently anxious to meet the needs of his readers, and not to presuppose unnecessarily an amount of knowledge on the part of the student that in most cases, unfortunately, is not at hand. It is to be regretted that the mass of material by which nowadays all editors of mathematical journals are overwhelmed tends more and more to make this pleasant manner of writing unfeasible.

Of minor details we may point out one that has already been emphasized by Herr von Mangoldt. In one of Beltrami's earlier papers (volume I, page 75) — where, judging from its title, nobody would expect such a thing — the chief properties of a figure were studied which, fourteen years later (viz., 1879), was rediscovered and fully discussed by C. Stéphanos. The figure is now generally known under the name of desmic configuration, given to it by the latter eminent geometer. Beltrami seems also to have been the first to determine the apparent size of a surface of the second degree. His paper on this subject (the fourth in the present edition) is dated 1863, and so precedes by twenty years the corresponding publication of H. A. Schwarz (Gesammelte Werke II, page 312).

In Beltrami's work geometrical ideas are prominent throughout. His method, however, was exclusively that of analysis. It would seem that the so-called pure geometry did not attract

him much. He certainly saw the traps into which contemporaneous writers had fallen and was fully aware of the limited range of the synthetic method.

For further information we refer the reader to a sketch of Beltrami's life-work by E. Pascal, published in the *Mathematische Annalen* (volume 57, 1903, pages 65–107), and thereby made generally accessible.

The editors, who modestly disappear behind their work, have taken great pains that the edition of the writings of their illustrious compatriot should appear in a dignified form. It will, for example, be difficult to find misprints. The paper is excellent, as also is the printing, which was done in the Tipografia Matematica di Palermo.

E. STUDY.

Études sur les Angles imaginaires. Par GEORGES DE LAPLANCHE. Paris, A. Hermann, 1908. 8vo. 135 pp. 3 francs.

THIS volume devotes considerable space to the development of the ordinary formulas, the computations, and the construction of graphs of trigonometric functions of complex numbers. The imaginary angle spoken of is merely the complex argument of these trigonometric functions, and in no way is it connected with the geometric conception of angle. De Moivre's formula is made the basis of all the developments, with no attempt at rigorous proof, while the graphical treatment rests on the usual Argand diagram. A gross misstatement is made on page 35 and repeated on page 36, to the effect that in $\sinh x$ and $\cosh x$ (written sh x, ch x), defined thus

$$\sinh x = \tfrac{1}{2}(e^x - e^{-x}), \quad \cosh x = \tfrac{1}{2}(e^x + e^{-x}),$$

the argument x is the length of the arc of an equilateral hyperbola, measured from its vertex. The equations of this hyperbola may be written

$$X = \cosh x, \quad Y = \sinh x,$$

whence

$$s = \int_0^x \sqrt{1 + 2\sinh^2 x}\, dx.$$

This is obviously not equal to x. The author seems to have

been misled in trying to extend the circular argument of cos x, sin x. This, it is true, may be taken as an arc, but it has long been known that in order to extend the analogy to the hyperbolic functions it is necessary to take as argument the ratio of the *sector* to one half the square of the radius. The baneful influence of considering trigonometric functions as lines depending on a linear argument is evidently not yet extinguished.

However, the author makes no use of his erroneous statement, but depends solely on series and De Moivre's formula, so that his formulas are correct enough.

The purpose of the work, scarcely realized in its treatment, is stated thus:

"Nous nous proposons de rechercher si le nombre imaginaire a des lignes trigonométriques: sinus, cosinus, tangentes, circulaires ou hyperboliques. . . . Nous en établissons la trigonométrie. Nous montrons de nouveaux moyens pour résoudre certains problèmes."

JAMES BYRNIE SHAW.

Vorlesungen über bestimmte Integrale und die Fourierschen Reihen. Von J. THOMAE. Leipzig, Teubner, 1908. 8vo. vi + 182 pp. 7.80 Marks.

This book undertakes to give a rather general view of the subjects mentioned in its title. It is somewhat more on the order of a descriptive course than either a systematic development or a practical handbook. Thus, in the first fifty-seven pages the student will see unfolded before him a view of the main theorems with some regard to the dangerous places near them. He will learn that there are such functions as Euler's $E(x)$, Dirichlet's function, Riemann's classic function (x), written here $r(x)$, Riemann's convergent series with an infinity of discontinuities

$$f(x) = r(x) + \frac{r(2x)}{2^2} + \frac{r(3x)}{3^2} + \cdots.$$

He will find that there is a definition of integral as the limit of a sum, which indeed is suggested by the inversion of a differentiation, and that under this definition many functions become integrable, for example those of Riemann mentioned above. The first mean value theorem he finds to hold equally for such functions, and the second mean value theorem is developed. He also finds that sometimes the variable may be changed,

sometimes not. Differentiation as to a parameter may be possible, but an example is given showing that this is not always the case. No indication is given of the conditions under which such differentiation is permissible.

The reader will be disappointed if he looks for a systematic treatment, even an elementary one. The classes of integrable functions are not clearly indicated, even if the two important ones are given. The tests for integrability of improper integrals are rather left to be surmised than clearly set forth; no really practical exemplification is given. The classes of functions which have improper integrals are not distinguished. In any particular case the student would feel that in a country known to have precipices he has neither guide nor map.

A systematic development of the properties of definite integrals, with the modifications necessary in improper integrals, is nowhere given; nor are the methods of evaluating forms more than hinted at by a few standard examples. The whole question of inversion of differentiation and integration, and of inversion of iterated integration is barely touched on. This is the more disappointing as the author says in his preface: "Aber als Hauptziel der Vorlesungen ist die wirkliche Auswertung bestimmter Integrale anzusehen."

As applications Fourier's series and Euler's integral are considered. Thirty-eight pages are devoted to Fourier's series, including twelve pages on vibrating cords. Illustrations are given of the determination of the coefficients for certain developments, among them the Riemann series mentioned above. The student is shown by examples that a series cannot always be integrated term by term, but is integrable term by term when uniformly convergent. It is shown that the sine series is differentiable term by term. What happens in the case of the cosine series is not mentioned. The conditions of convergence of Dirichlet are proved, and Schwarz's example is given to show that continuity merely is not a sufficient condition. Other questions of convergence, particularly uniform convergence, are barely hinted at, so that the matter of integrability is left open. No attempt is made to set forth the modern widening of Dirichlet's conditions.

The only example given is the solution of the equation of a vibrating string

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}.$$

The usual particular cases are stated and rather fully discussed, in connection with musical strings.

Again the student desiring more than a cursory view of the subject would be disappointed. The development of the cosine series is awkward. The essential difference between representing x, say by a sine series, or a cosine series, or a mixed series, is not clearly exhibited. This is always a point of confusion to the student. The manner in which the various approximation curves give the graph of the function is not intimated. The classes of developable functions are left under the narrow Dirichlet conditions. No mention is made of Fourier's classic developments in the theory of heat, where indeed the really practical value of Fourier series shows at its best.

Twenty-eight pages are devoted to double integrals, followed by ten pages on Fourier's integrals. The practical value of the latter is barely mentioned. Eighteen pages are given to the elementary formulas of the B and the Γ functions.

Thirty-two pages are given to the integration of expressions in two independent variables,

$$dw = pdx + qdy.$$

This introduces naturally the functions of a complex variable. It is shown that these lead to solutions of Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

It is regrettable that no modern text-book exists which presents the subject of definite integrals and their applications in a complete and practical way. Of course treatises on the theory of functions of a real variable usually contain the general theory, but they also contain much extraneous matter. With the growing use and importance of harmonic analysis particularly, such a text becomes more necessary. The present volume, interesting and quite smoothly carried out as it is, does not meet this demand. It is for the general reader rather than the student who desires to become skilful with these useful tools. Students of mathematical physics must look elsewhere for what they want.

JAMES BYRNIE SHAW.

NOTES.

THE annual meeting of the AMERICAN MATHEMATICAL SOCIETY will be held in the Rogers Building of the Massachusetts Institute of Technology, 491 Boylston Street, Boston, opening on Tuesday afternoon, December 28. Wednesday morning will be devoted to a joint session with Section A of the American Association. Arrangements have been made for a joint smoker with the Association of Mathematical Teachers in New England on Tuesday evening, and for the usual dinner of the Society on Wednesday evening.

THE Annual Register of the AMERICAN MATHEMATICAL SOCIETY is now in preparation and will be issued in January. Blanks for furnishing necessary information have been sent to the members. Early notice of any changes since the issue of the last Register will greatly facilitate the work of the Secretary. The Register is widely circulated and it is desirable that the information which it contains should be accurate and reliable.

THE twenty-fifth regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY will be held at the University of Chicago on Friday, December 31, 1909, and Saturday, January 1, 1910. Titles and abstracts of papers to be presented at this meeting should be in the hands of the Secretary of the Section on or before December 10, 1909. It is desired that all abstracts be printed in advance of the meeting.

THE seventeenth regular meeting of the San Francisco Section will be held at Stanford University, February 26, 1910.

THE concluding (October) number of volume 31 of the *American Journal of Mathematics* contains the following papers: "The asymptotic representation of the elliptic cylinder functions," by W. MARSHALL; "A theory of invariants," by L. E. DICKSON; "Symmetric binary forms and involutions (continued)," by A. B. COBLE; "A theory of geometrical relations," by A. R. SCHWEITZER.

UNIVERSITY OF PARIS. The following mathematical courses are announced for the semester beginning November 3, 1909: —By Professor G. DARBOUX: Infinitesimal geometry, particu-

larly triply orthogonal systems, two hours. — By Professor E. GOURSAT: Differential and integral calculus, elements of the theory of analytic functions, two hours. — By Professor L. RAFFY: Twisted curves, properties of curves traced on surfaces, one hour. — By Professor E. BOREL: Definite integrals, with certain applications, two hours. — By Professor P. PAINLEVÉ: General laws of equilibrium and motion, two hours. — By Professor P. APPELL and Mr. E. BLUTEL: General mathematics, first part, one hour. — By Professor H. POINCARÉ: Movements of the heavenly bodies around their center of gravity, two hours. — By Professor J. BOUSSINESQ: Mechanical theory of light, two hours.

In the Ecole Normale. By Professor J. TANNERY: Differential and integral calculus. — By Professor L. RAFFY: Applications of analysis to geometry. — By Professor E. BOREL: Mathematics. — By Professor J. HADAMARD: Mathematics.

Weekly conferences will be held by Professor RAFFY and Messrs. CARTAN, BLUTEL, and SERVANT.

For the second semester the following courses are announced: — By Professor E. PICARD: The principal developments in series occurring in mathematical physics. — By Professor E. GOURSAT: Differential equations, partial differential equations. — By Professor P. PAINLEVÉ: General laws of motion of systems, analytic mechanics, hydrostatics and hydrodynamics. — By Professor P. APPELL: Analysis and mechanics. — By Professor J. BOUSSINESQ: Theory of light. — By Professor G. KOENIGS: General theory of mechanisms.

In the Ecole Normale. By Professor J. TANNERY: Differential and integral calculus. — By Professor E. BOREL: Mechanics. — By Professor J. HADAMARD: Mathematics.

THE president of the Euler commission, Professor F. Rudio, reported at the Salzburg meeting of the Deutsche Mathematiker-Vereinigung that the sixth international congress for the science of insurance and the association of German engineers had each contributed five thousand francs toward the publication of the works of Euler. Subscriptions have now been received for more than 275 sets, and in addition cash contributions of more than 130,000 fr. have been acknowledged. This sum has been conservatively invested, and it is estimated that it will earn at least 30,000 fr. before it need be paid out. The total assets are now over 450,000 fr., and a larger edition than was originally planned (400 sets) will probably be undertaken.

THE index of Euler's writings, prepared by Dr. G. ENESTRÖM, is now in press, and will appear in the spring of 1910 as a supplementary volume of the *Jahresbericht der deutschen Mathematiker-Vereinigung*. An appendix to volume 10 of the *Jahresbericht*, containing additions and an index to Professor BURKHARDT's report on oscillating functions, is also in preparation.

THE first Scandinavian congress of mathematicians was held at Stockholm, September 22–25, 1909, under the presidency of Professor G. MITTAG-LEFFLER. The following papers were read: By A. C. BJERKNES, "Mathematical investigation of meteorological problems"; by I. FREDHOLM, "On integral equations"; by HJELMSLER, "Some geometric principles"; by H. v. KOCH, "Systems of equations with an infinite number of variables"; by E. LINDELÖF, "Picard's theorem in the theory of functions"; by G. MITTAG-LEFFLER, "Arithmetic formulation of the theory of functions"; by E. PHRAGMÉN, "Theory of integral functions of the second order"; by C. STÖRMER, "Mathematical treatment of polar and magnetic storms"; by SUNDMAN, "Real singularities in the problem of three bodies"; by H. G. ZEUTHEN, "Some methods in the texts on geometry"; by K. BIRKELAND, "On irregular integrals of linear differential equations"; by A. C. BJERKNES, "On a modified form of the Maxwell lines"; by BOHR, "On Dirichlet's summation formula, particularly on the boundary of the region of convergence"; by F. BRODÉN, On the so-called Richard paradoxes"; by BUCHT, "Reduction of cyclic to metacyclic equations"; by CHARLIER, "Mathematical desiderata in the theory of probabilities"; by EKMAN, "Questions of stability in hydrodynamics"; by P. HANSEN, "On Taylor's series"; by HESSELBERG, "An application of the lines of hydrodynamic continuity to the determination of the vertical motion of the atmosphere"; by E. A. HOLMGREN, "On systems of linear partial differential equations with real characteristics"; by C. JUEL, "Note on a non-analytic surface of revolution"; by LINDEBERG, "On the proper name for the so-called Weierstrass functions in the calculus of variations"; by MELLIN, "A uniform theory for gamma functions and hypergeometric functions"; by J. MOLLERUP, "Regions of convergence for orthogonal functions"; by NÖRLUND, "Linear difference equations"; by C. W. OSEEN, "A partial differential equation in mathematical physics"; by A. PALMSTROM, "On cyclic numbers"; by STRIDSBERG, "Some arithmetic properties of certain

transcendental functions," and "A study of the arithmetic properties of the integrals of algebraic differential equations"; by A. THUE, "Some new arithmetic properties of algebraic numbers with an application to Diophantine equations"; by WICKSELL, "Mathematical deduction of investments"; by VON ZEIPEL, "On secular perturbations of comets."

THE association of Swiss mathematical teachers held its eleventh meeting at Solothurn, October 10, 1909. Four formal papers were read, reports were heard and plans adopted regarding the activity of the international committee on mathematical instruction, and a committee of fifteen was appointed to consider and report on a general plan for vacation courses in mathematics.

AT the meeting of the New York section of the Association of teachers of mathematics in the middle states and Maryland held on November 12 the following papers were read: By E. R. von NARDROFF, "Mathematics for service"; by W. H. METZLER, "Mathematics for training and culture." Two meetings will be held during the first half of the year 1910.

THE committee of fifteen appointed by the Central association to consider the teaching of geometry has nearly completed its report, and expects to present it during the winter.

DR. A. ADLER, of the technical school at Vienna, has been promoted to an associate professorship of mathematics.

PROFESSOR P. STÄCKEL, of the technical school at Hanover, and Dr. G. ENESTRÖM, librarian of the University of Stockholm, have been elected honorary members of the society of Swiss naturalists.

DR. G. FABER, of the technical school at Karlsruhe, has been appointed associate professor of mathematics at the University of Tübingen.

AN honorary doctorate of philosophy has been conferred by the University of Leipzig upon Professor I. FREDHOLM, of the University of Stockholm.

DR. E. HILB, of the University of Erlangen, has been appointed associate professor of mathematics at the University of Würzburg.

PROFESSOR F. KLEIN, of the University of Göttingen, has been elected chairman of the board of directors of the German museum at Munich.

PROFESSOR R. LEHMANN-FILHÉS, of the University of Berlin, has been appointed honorary professor of mathematics.

PROFESSOR C. NEUMANN, of the University of Leipzig, has received the title of Geheimrat.

THE University of Heidelberg recently publicly celebrated the eightieth birthday of Professor MORITZ CANTOR, who was born at Mannheim, August 23, 1829.

THE Royal Society has awarded its Copley medal to Dr. G. W. HILL.

PROFESSOR G. E. FISHER, of the University of Pennsylvania, has been appointed Dean of the College of Arts and Sciences.

AT the inauguration of President Bryan, of Colgate University, the degree of doctor of science was conferred upon Professor F. C. FERRY, of Williams College.

PROFESSOR R. J. ALEY, of the University of Indiana, has been appointed Superintendent of Public Instruction of the State of Indiana.

AT the University of Pennsylvania Professor I. J. SCHWATT has been promoted to a full professorship of mathematics.

AT Purdue University Professor C. H. BECKETT has been promoted to an associate professorship of mathematics.

AT Oberlin College Miss M. E. SINCLAIR has been promoted to an associate professorship of mathematics.

DR. C. A. MOORE, of the University of Cincinnati, has been promoted to an assistant professorship of mathematics.

AT Syracuse University Professor W. G. BULLARD has been promoted to a full professorship of mathematics.

AT the University of Michigan Dr. J. W. BRADSHAW has been promoted to an assistant professorship of mathematics.

AT the University of Nebraska Dr. W. C. BRENKE has been promoted to an associate professorship of mathematics.

AT Wesleyan University Mr. B. H. CAMP has been promoted to an associate professorship of mathematics.

MR. A. F. CARPENTER has been appointed instructor in mathematics at the University of Washington.

AT Stevens Institute Mr. R. F. DEIMEL has been promoted to an assistant professorship of mathematics.

AT Indiana University Dr. U. S. HANNA has been promoted to an associate professorship of mathematics.

AT the University of Pennsylvania Professor I. J. SCHWATT has been promoted to a full professorship of mathematics.

DR. CLARA E. SMITH has been appointed instructor in mathematics at Wellesley College.

DR. RUTH G. WOOD, of Smith College, has been promoted to an associate professorship of mathematics.

PROFESSOR J. C. STONE, of the Michigan State Normal College, has been appointed head of the department of mathematics at the New Jersey State Normal School, Montclair, N. J.

MR. F. C. MOORE has been appointed assistant professor of mathematics at the New Hampshire College of Agriculture and Mechanic Arts.

DR. H. L. SLOBIN has been appointed assistant in mathematics at the University of Minnesota.

DR. O. DUNKEL, of the University of Missouri, has been granted leave of absence to study in Europe.

DR. M. O. TRIPP, of the College of the City of New York, has been granted leave of absence for the present academic year, which he will spend abroad.

PROFESSOR R. D. CARMICHAEL, of the Alabama Presbyterian College for Men, is spending a year's leave of absence in study at Princeton University.

PROFESSOR B. F. YANNEY, of Mount Union College, is spending a year's leave of absence in study at the University of Chicago.

PROFESSOR J. H. VAN AMRINGE, head of the department of mathematics in Columbia University, and dean of the college, will retire from active service at the end of the present academic year, when he will have completed fifty years of service in the university and reached his seventy-fifth birthday.

PROFESSOR HUGH BLACKBURN, emeritus professor of mathematics at the University of Glasgow, died October 9, 1909, at the age of 86 years. He held the chair of mathematics from 1849 to 1879.

MAJOR GENERAL O. O. HOWARD, U. S. A., who died October 26, 1909, at the age of 78 years, was associate professor of mathematics at the Military Academy at West Point from 1857 to 1861.

CATALOGUES of second-hand mathematical books: Gustav Fock, Schlossgasse 7, Leipzig, catalogue no. 361, 2,600 titles in pure and applied mathematics. — Koehler's Antiquarium, Kurprinzstrasse 6, Leipzig, catalogue no. 581, about 1,100 titles in history, encyclopedia, and bibliography of exact sciences; catalogue no. 582, about 3,800 titles in pure and applied mathematics. — A. Hermann, rue de la Sorbonne 6, Paris, catalogue no. 99, 1,767 titles in pure and applied mathematics. — Basler Buch- und Antiquariatshandlung, catalogue no. 330, 460 titles in pure and applied mathematics.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

AMSON (E.). Eine zeichnerisch wohl verwertbare Konstruktionsweise des allgemeinen Kegelschnittes. (Progr.) München, 1909. 8vo. 19 pp.

AQUINO (L. D'). Nota sulla teoria degli integratori. Napoli, D'Auria, 1909. 8vo. 16 pp.

BEUTEL (E.). Algebraische Kurven. Teil I: Kurvendiskussion. Leipzig, 1909. 8vo. 147 pp. Cloth. M. 0.80

BEZLOJA (A.). Das Schliessungsproblem. (Progr.) M. — Weisskirchen, 1908. 8vo. 16 pp.

BIANCHI (L.). Lezioni di geometria differenziale. Vol. III: Teoria delle trasformazioni delle superficie applicabili sulle quadriche. Pisa, Spoerri, 1909. 8vo. 5 + 350 pp. L. 15.00

BLAINE (R. G.). The calculus and its applications. A practical treatise. London, Constable, 1909. 8vo. 332 pp. Cloth. 4s. 6d.

BÜCHER, neue, über Naturwissenschaften und Mathematik. (Die Neuigkeiten des deutschen Buchhandels nach Wissenschaften geordnet.) Mitgeteilt Sommer 1909. Leipzig, Hinrich, 1909. 8vo. Pp. 25–46. M. 0.30

BURALI-FORTI (C.) e MARCOLONGO (R.). Elementi di calcolo vettoriale, con numerose applicazioni alla geometria, alla meccanica e alla fisica matematica. Bologna, Zanichelli, 1909. 8vo. 5 + 174 pp. L. 5.00

——. Omografie vettoriali, con applicazioni alle derivate rispetto ad un punto ed alla fisica-mathematica. Torino, Petrini, 1909. 8vo. 11 + 115 pp. L. 4.00

CAIN (W.). A brief course in the calculus. 2nd edition. London, Blackie, 1909. 8vo. Cloth. 6s.

CHATELAIN (E.). Relations entre les nombres de classes dans les différents ordres de formes binaires quadratiques d'un déterminant donné. (Diss.) Zurich, 1908. 8vo. 82 pp.

ENGEL (F.). Hermann Grassmann. Leipzig, 1909. 8vo. 15 pp. M. 0.80

EISENHART (L. P.). A treatise on the differential geometry of curves and surfaces. Boston, Ginn & Co., 1909. 8vo. 11 + 474 pp. Cloth. $4.50.

FOUËT (E. A.). Leçons élémentaires sur la théorie des fonctions analytiques. 2e édition, entièrement refondue. Vol. 2: les fonctions algébriques; les séries simples et multiples; les intégrales. Paris, Gauthier-Villars, 1910. 8vo. 11 + 267 pp. Fr. 9.00

FRANKENBACH (F. W.). Lineare Erzeugung der Kegelschnitte und auf ihr beruhende Ableitung der Kegelschnittsgleichungen. Beitrag zur Lehre von den Kurven 2ter Ordnung. Liegnitz, 1909. 8vo. 49 pp. M. 1.00

FRICKE (R.). Hauptsätze der Differential- und Integral-Rechnung, als Leitfaden zum Gebrauch bei Vorlesungen zusammengestellt. 5te Auflage. Braunschweig, Vieweg, 1909. 8vo. 15 + 219 pp. M. 5.80

GOEDE (G. DE). Vollständige Lösung des grossen Fermatschen Satzes. Antwort auf das Preisausschreiben der königlichen Gesellschaft der Wissenschaften zu Göttingen von 1908. Deventer (Niederlande), Dixon, 1909. 8vo. 11 pp. M. 0.40

GÜNTZEL (F.). Ueber Gruppierungen und Realitätsverhältnisse gewisser Punkte bei Raumkurven vierter Ordnung erster Species. (Diss.) Jena, 1909. 8vo. 62 pp.

HAUSSNER (R.). See SCHERING (E.).

HEIDELBERG (P.). Allgemeiner Beweis des grossen Fermatschen Satzes. Wiesbaden, Staadt, 1909. 8vo. 3 pp. M. 0.80

KARNASCH. Beweis für den Fermatschen Satz, dass die Gleichung $x^n + y^n = z^n$ in ganzen Zahlen nicht lösbar ist, wenn $n > 2$. Berlin, Mayer, 1909. 8vo. 7 pp. M. 0.80

KLEINSCHMIDT (M.). Elementarer Beweis des Fermatschen Satzes. Rostock, Boldt, 1909. 8vo. 6 pp. M. 1.50

KLOBASA (C.). Ueber parallelepipedische Zahlen. (Progr.) Troppau, 1909. 8vo. 6 pp.

LEBON (E.). Henri Poincaré. Biographie, bibliographie analytique des écrits. Paris, Gauthier-Villars, 1909. 8vo. 8 + 80 pp. Fr. 7.00

LICHTENSTEIN (L.). Zur Theorie der gewöhnlichen Differentialgleichungen und der partiellen Differentialgleichungen zweiter Ordnung. Die Lösungen als Funktionen der Randwerte und der Parameter. (Diss.) Berlin, 1909. 8vo. 40 pp.

LOVE (A. E. H.). Elements of the differential and integral calculus. Cambridge, University Press, 1909. 8vo. 222 pp. Cloth. 5 s.

MARCOLONGO (R.). See BURALI-FORTI (C.).

MANTZ (O.). Ebene Inversionsgeometrie. (Progr.) Basel, 1909. 4to. 62 pp.

MEYER (C.). Zur Theorie des logarithmischen Potentials. Berlin, 1909. 8vo. 68 pp. M. 2.00

NOETHER (F.). Ueber rollende Bewegung einer Kugel auf Rotationsflächen. (Diss., Munich.) Leipzig, Teubner, 1909.

PAULY (J.). Notions élémentaires du calcul différentiel et du calcul intégral, 2e edition. Paris, 1909. 8vo. Fr. 6.80

PICKFORD (A. G.). Elementary projective geometry. Cambridge, University Press, 1909. 8vo. 268 pp. Cloth. 4s.

PUCCINI (A.). Le configurazioni piane regolari d'indice 3. Padova, Prosperini, 1909. 8vo. 65 pp.

SCHEFFERS (G.). See SERRET (J. A.).

SCHERING (E.). Gesammelte mathematische Werke. Herausgegeben von R. Haussner und K. Schering. 2ter (Schluss-) Band. Berlin, Mayer, 1909. 8vo. 8 + 472 pp. M. 25.00

SCHERING (K.). See SCHERING (E.).

SCHMALL (C. U.). A first course in analytic geometry. London, Blackie, 1909. 8vo. Cloth. 6s.

SCHMIDT (J.). Der Infinitesimalkalkül. (Progr.) Wien, 1909. 8vo. 15 pp.

SCHUR (F.). Grundlagen der Geometrie. Leipzig, Teubner, 1909. 8vo. 10 + 192 pp. M. 7.00

SERRET (J. A.). Lehrbuch der Differential- und Integralrechnung. Nach Axel Harnacks Uebersetzung. 3te Auflage. Neu bearbeitet von G. Scheffers. 3ter (Schluss-) Band. Differentialgleichungen und Variationsrechnung. Leipzig, Teubner, 1909. 8vo. 12 + 658 pp. Cloth. M. 13.00

SMITH (D. E.). Rara Arithmetica. Second edition. Boston, Ginn & Co., 1909. Large 8vo. 16 + 507 pp. Cloth. $4.50

TEIXEIRA (J. G.). Obras sobre matematica publicadas por ordem do governo Portuguès. Vol. V: Traité des courbes spéciales remarquables planes et gauches. Traduit de l'Espagnol, revu et très augmenté. Coimbra, 1909. 4to. 497 pp. M. 16.00

THAER (C.). Eine Ausdehnung der Galoisschen Theorie auf algebraische Gleichungen mit mehrfachen Wurzeln. (Habilitationsschrift.) Jena, 1909. 8vo. 36 pp.

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VILLAFANE Y VIÑALS (J. M.). Tratado de análisis matemático (algebra superior). 3a parte; Teoría general de ecuaciones. Barcelona, Caridad, 1909. 284 pp. P. 10.00

VOGT (W.). Synthetische Theorie der Cliffordschen Parallelen und der linearen Linienörter des elliptischen Raumes. (Habilitationsschrift.) Karlsruhe, 1909. 8vo. 8 + 58 pp.

II. ELEMENTARY MATHEMATICS.

AMODEO (F.). Complementi di analisi algebrica elementare, con appendice sulle sezioni coniche. Parte II del Vol. II degli *Elementi di matematica*, ad uso del 20 biennio degli istituti tecnici. Napoli, Pierro, 1909. 8vo. 11 + 284 + 28 pp. L. 3.00

AUBRY. See BALL (W. R.).

BALL (W. R.). Récréations mathématiques et problèmes des temps anciens et modernes. 2e édition française, traduite d'après la 4e édition anglaise et enrichie de nombreuses additions par J. Fitz-Patrick. 3e partie, avec addition de MM. Margossian, Reinhart, Fitz-Patrick et Aubry. Paris, Hermann, 1909. 16mo. 370 pp. Fr. 5.00

BEHRENDSEN (O.) und GÖTTING (E.). Lehrbuch der Mathematik nach modernen Grundsätzen. Ausgabe für höhere Mädchenlehranstalten, zugleich Unterstufe für Lyzeen und Studienanstalten. Leipzig, Teubner, 1909. 8vo. 7 + 310 pp. M. 3.00

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BOURLET (C.). Eléments d'algèbre, contenant 631 exercices et problèmes, rédigés conformément aux programmes de l'enseignement secondaire 1er et 2e cycles, classes de troisième A, seconde et première A et B de l'enseignement secondaire et aux programmes de l'enseignement primaire supérieur. 6e édition, revue. Paris, Hachette, 1909. 16mo. 314 pp. Fr. 2.00

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FRENZEL (C.). See MEHLER (F. G.).

GÖTTING (E.). See BEHRENDSEN (O.).

HENRICI (J.) und TREUTLEIN (P.). Lehrbuch der Elementar-Geometrie 1ter Teil. Gleichheit der Gebilde in einer Ebene und deren Abbildung ohne Massänderung (nebst einer Aufgabensammlung). 4te Auflage. Leipzig, Teubner, 1909. 8vo. 8 + 139 pp. M. 2.40

HOČEVAR (F.). Lehr- und Uebungsbuch der Arithmetik für Gymnasien, Realgymnasien und Realschulen. Unterstufe (I, II und III Klasse). 7te nach den neuen Lehrplänen umgearbeitete Auflage. Vienna, Tempsky, 1909. Kr. 2.10

JADANZA (N.). Trattato di geometria pratica. Dispensa 31–50 (fine). Torino, Bona, 1909. 8vo. Pp. 481–796.

LESSER (O.). Lehr- und Uebungsbuch für Unterricht in der Arithmetik und Algebra. Teil II. Ausgabe A: Für die oberen Klassen der Realanstalten. Vienna, Tempsky, 1909. M. 3.00

——. See SCHWAB (K.).

MAHLERT (A.). See MÜLLER (H.).

MANUEL d'algèbre et de trigonométrie. Paris, Poussielgue, 1909. 16mo. 8 + 216 pp.

MARGOSSIAN. See BALL (W. R.).

MATRICULATION model answers: mathematics. Being the London University matriculation papers in mathematics from June 1906 to June 1909. London, Clive, 1909. 8vo. 160 pp. 2s.

MATTINA (P.). Aritmetica goniometrica: introduzione alla trigonometria. Palermo, Virzi, 1909. 8vo. 15 + 273 pp. L. 5.00

MEHLER (F. G.). Hauptsätze der Elementar-Mathematik zum Gebrauche an höheren Lehranstalten. Bearbeitet von A. Schulte-Tigges. Ausgabe B. Oberstufe, 2ter Teil: Arithmetik mit Einschluss der niederen Analysis, Trigonometrie und Stereometrie. Für die oberen Klassen höherer Lehranstalten bearbeitet unter Mitwirkung von C. Frenzel. Berlin, Reimer, 1909. 8vo. 7 + 169 pp. M. 1.50

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MÜLLER (H.) und MAHLERT (A.). Lehr- und Uebungsbuch der Arithmetik und Algebra für Studienanstalten. Ausgabe B: Für Oberrealschul- und realgymnasiale Kurse. 1ter Teil: Bis zur Lehraufgabe der Klasse IV. Leipzig, Teubner, 1909. 8vo. 6 + 192 pp. M. 2.40

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NATHING (A.). See BELIKOW (A.).

OXFORD local examinations. Papers set in arithmetic and mathematics at the senior, junior and preliminary examinations held in March and July, 1909. London, Parker, 1909. 8vo. 8d.

PLATH (J.). Lehrbuch der Mathematik zur Vorbereitung auf die Mittelschullehrer-Prüfung und auf das Abiturientenexamen am Realgymnasium. Im Anschluss an die Baltin-Maiwaldsche Seminarausgabe des Müllerschen Lehrbuches und in Verbindung mit Prof. Müller für den Selbstunterricht bearbeitet. 2te Auflage. Leipzig, Teubner, 1909. 8vo. 8 + 272 pp. M. 4.00

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RADFORD (E. M.). Exercise papers in elementary algebra. London, Dent, 1909. 12mo. 120 pp. 2 s.

RATINET (A.). See BOUVART (C.).

REINHART. See BALL (W. R.).

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SCHULTE-TIGGES (A.). See MEHLER (F. G.).

SCHWAB (K.) und LESSER (O.). Mathematisches Unterrichtswerk zum Gebrauche an höheren Lehranstalten. Im Sinne der Meraner Lehrpläne bearbeitet. Vol. I: Lehr- und Uebungsbuch für den Unterricht in der Arithmetik und Algebra. Wien, Tempsky, 1909. 8vo.
1ter Teil, 203 pp. M. 2.80
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SMITH (D. E.). The Teaching of Arithmetic. New York, Columbia University Press, 1909. 120 pp. $0.75

SUPPANTSCHITSCH (R.). Grundriss der Geometrie für Gymnasien und Realgymnasien. Heft II, für die IIIte Klasse. Wien, Tempsky. 1909. Kr. 1.70

THIEME (H.). Leitfaden der Mathematik für Realanstalten. 1ter Teil: Die Unterstufe. 4te Auflage. Leipzig, Freytag, 1909. 8vo. 135 pp. M. 1.80

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III. APPLIED MATHEMATICS.

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LOZANO (E.). See BROWN (H. T.).

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